

# NON-LINEAR ANALYSIS OF COMPOSITE BEAMS WITH MINIMAL MODELLING AND CALCULATION EFFORT FOR STRAIN-LIMITED DESIGN

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**Abstract.** *In the following, an analysis method is presented that enables the modelling of composite beams in commercial programs with minimum effort, considering non-linear material behaviour and nonlinear flexibility of the composite joint. A particular focus is placed on the deformations, especially those of the composite joint in horizontal and vertical directions. A detailed insight into the necessary accuracy of the material modelling is presented. The model is subsequently verified by means of comparative analyses of real beam tests.*

## 1 INTRODUCTION

The construction industry has an important contribution to limiting global warming through efficient load-bearing structures and the associated savings in raw materials. However, the safety level of the structures must not suffer as a result. The effectiveness of composite beams made of concrete and steel is beside the performance of both composite materials mainly characterised by the load bearing and deformation behaviour of the shear connectors. In order to enable further developments with regard to safety and economic efficiency, calculation methods are required which, in addition to the load-bearing capacity, also consider the deformations of the beam and the deformability of the shear connectors.

In the current standard of EN 1994-1-1 [1], for the strain-limited design it is assumed that the total cross-section is kept plane, i.e. the flexibility of the composite joint is neglected. Since this flexibility of the composite joint, especially in beams with low shear connection ratio, plays a decisive role on the load-bearing and deformation behaviour of composite beams, an analysis model is presented which allows the realistic modelling of composite beams by using software that is frequently used in structural design offices. In the model the non-linear material behaviour of steel and concrete as well as a non-linear load-slip behaviour of the composite joint is considered.

In addition to the strain-limited design, for which detailed investigations regarding reliability are still required, the analysis model is already suitable for use in research and could be a very suitable complement to complex scientific programmes. For example, post-calculations of test results as well as investigations of the influence of individual parameters on composite beams are very easily possible and with high accuracy. By using commercial programs, adjustments in the model, for example a change of the positions of the shear connectors, as well as the direct output of the results, such as strains and internal forces, are much more easily possible. Moreover, the use of a high-performance computer is not necessary here.

In the following, the structure of the model is explained first. Subsequently, the material modelling and the influence on the results are described. To verify the models, comparative analyses of real beam tests and the results of numerical simulations are presented. Based on the analysis model, parameter studies on the influence of the load-deformation curve of the beam elements are presented.

## 2 GENERAL

In addition to the Gamma-Method, framework models have long been the standard method in timber construction for the calculation of composite timber cross-sections or timber-concrete composite structures. Due to the almost linear elastic material behaviour of timber, a linear elastic analysis in timber engineering normally is sufficient [2, 3, 4, 5]. In steel-concrete composite construction, due to the high ductility of the composite beams, consideration of the non-linear material behaviour is essential for modelling real experiments and for economic design. For this reason, framework models have only been used to a limited extent in composite constructions made out of steel and concrete.

However, due to increasingly powerful commercial programs for structural design and increasing computer performance, composite beams can now be analysed with framework models considering the non-linear material behaviour of the composite members as well as the non-linear load-slip relation of the composite joint. That means little modelling and computing effort compared to scientific numerical simulations. Moreover, the method can easily be adopted for practically relevant design problems due to the use of common engineering programmes.

As mentioned, the calculation model is a two-dimensional framework structure in which the upper chord corresponds to the concrete slab and the lower belt chord to the steel profile. The beams are assigned the respective cross-sectional properties of the real composite components. For the steel profiles, it is usually possible to use profile databases. Since the programme does not consider load propagation perpendicular to the model plane, it is essential to consider the effective widths of the concrete slab according to EN 1994-1-1 [1]. Since the calculation effort increases enormously due to the resulting trapezoidal concrete cross-sections, the effective width also can be modelled with areas of constant width. The reinforcement layer is modelled as one rectangular bar with the diameter of the reinforcement bar as height and a width resulting from the total area of the reinforcement laying in the effective width of the concrete. The reinforcement is placed on the axis of the concrete cross-section and the exact height is set by using a bar eccentricity. As usual in numerical simulations, a rigid bond between reinforcement and concrete is assumed. Based on the framework model given in [6] a horizontal and a vertical spring is arranged in the composite joint in place of each connector. These springs are connected to the steel beam and the concrete slab with vertical rigid bars. The beam elements lay on the centroid of the concrete slab and the steel profile. The model is shown in figure 1. Even though the present study only focuses on headed studs as shear connectors, if the load and deformation behaviour of other shear connectors are available, these can also be implemented in model.

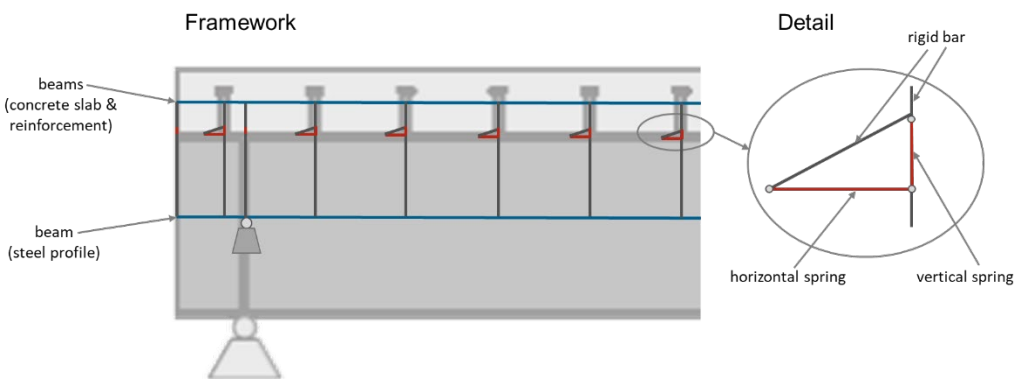


Figure 1. Structure of the framework model

As a simplified alternative to the framework model shown in figure 1, the springs and the inclined rigid bar could be replaced by hinges. Specifically, a full moment hinge, a normal force hinge with the vertical stiffness of the composite joint and a shear force hinge with the horizontal composite joint would be

adopted. However, since in further researches the influence of the slip on the uplift is to be modelled via an inclined position of the "horizontal" spring, the set-up from figure 1 is preferred. The magnitude of the uplift can be specified via the angle of the spring, so research is currently still being carried out into a general parameter for the position.

In addition to the framework models, in which the concrete slab and steel profile are modelled as a beam section, shell models were also investigated. Here, the concrete slab is modelled as one shell and the steel profile as three shells, separated for the flanges and the web. Using shell elements, the strains and curvatures of concrete and steel could be modelled a more precise, especially locally. In contrast, the determination of the internal forces of the composite materials proved to be disadvantageous. While the internal forces of beam elements can be computed directly, the internal forces of shells must be determined either by manually setting sections in the programme or by integrating the stresses over the cross-section.

For the beams investigated here, steel profile with symmetrical cross-section and small ratio of plastic to elastic bending resistance, both methods achieve similar results. For general cross-sections, the more complex method using shell-elements seems to provide the more precise results. Since the modelling and calculation effort of the shell models is significantly higher, this paper will concentrate only on the framework models out of beam elements.

### 3 MATERIAL

If a powerful commercial software such as RFEM [7] from Dlubal is used to analyse the framework models, the material behaviour of the components is defined very detailed in tabular form so that plastic effects of the materials as well as cracking of concrete can be considered.

Based on results from push-out tests, simplified load-slip relationships of the shear connectors are determined. This simplified relationship is used in the model as stiffness of the horizontal springs. Figure 2 shows on the left the results of push-out tests of various headed studs with different concrete strengths from [8] and the simplified load-slip relationship of one headed stud on the right. To determine the horizontal spring stiffness for the model, numerous results of push-out tests of different types of shear connectors and concrete strengths can be integrated from various literature sources.

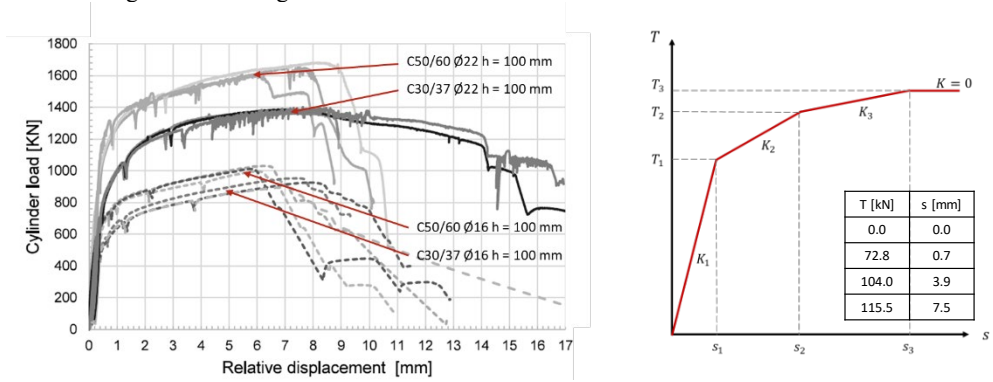


Figure 2. left: load-slip-relationship of headed stud from push-out tests from [8]; right: simplified load-slip relationship

Unlike the horizontal springs, the vertical springs in the model are defined separately for tensile and compressive forces. Since in the real structure the vertical compression forces are transmitted between concrete and steel via contact of both components, the vertical spring stiffness for compression forces is assumed to be almost unlimited rigid. In a model to determine the vertical stiffness of composite joints of headed studs proposed in [9] the stiffness of the concrete surrounded by the headed stud is applied while the headed stud is assumed rigid. First an equivalent circular concrete surface is determined. Due to the

small number of available test results for headed studs under axial tensile forces specifying the connections stiffness, the equation for determining the equivalent concrete diameter is only calibrated on one test series. For the beams investigated below, consisting of a C30/37 concrete and 16 mm diameter headed stud, numerical simulations have shown that a major part of the uplift results from the elongation of the bolt shaft and that the concrete compression only has a minor contribution. For this reason, the tensile stiffness of the bolt shaft is modelled as vertical spring stiffness under tensile load. This assumption is also frequently made in fastening technology. Since the distribution of uplift varies depending on the stiffness of the connector and the concrete, further investigations are necessary to enable general statements. A serial arrangement of the bolt stiffness and the stiffness of the surrounding concrete is aimed. However, in order to be able to determine the stiffness of the concrete with confidence, further investigations are necessary to apply the model from [9] to general situations. Comparative calculations show that the vertical spring stiffness only influences the size of the uplift, but not the other results. If the uplift is not of interest, as it is usually the case, a rigid connection can be used in vertical direction. In the model, rigid bars with only vertical springs are arranged in the composite joint at the supports as well as under concentrated loads, comparable to load introduction stiffeners in real steel profiles. If no connectors are arranged at these points, the springs only transmit compressive forces.

Figure 3 shows the material models used for steel, concrete and the reinforcement. To consider the real load-bearing-behaviour of the composite beams the analysis is done with the average of the tested material properties. The stress-strain behaviour of the steel was modelled from data of tensile tests with consideration of the yield plateau and resolidification. The stress-strain behaviour of the concrete in compression was modelled parabolic according to EN 1992-1-1 [10] without modelling the descending range using the test results of the average compressive strength and the modulus of elasticity. Since it is not possible to consider the falling branch in the stress-strain-curve after reaching the compressive strength in the programme, the graph is constant up to a maximum strain  $\epsilon_{cu,1}$  of 3.5‰. This results in a slight overestimation of the concrete at higher compression strains. The concrete under tensile forces is modelled linearly elastic up to the tensile strength. In the programme, the function "crack" is activated in the material modelling, so that stress drops after once reaching the tensile strength.

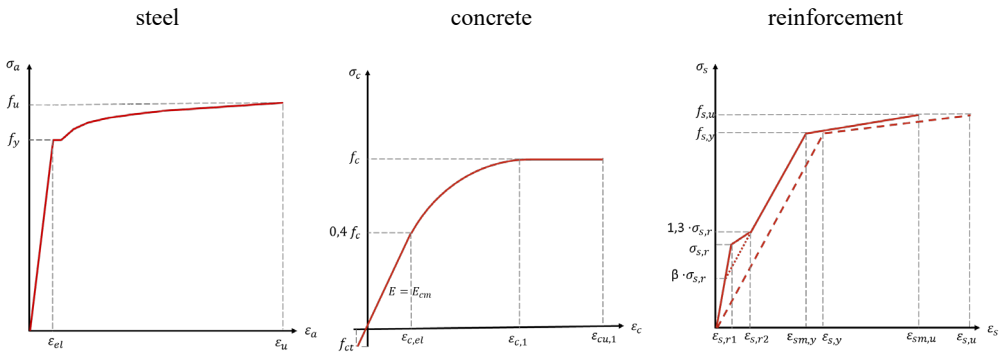


Figure 3. Stress-strain relationships used for steel, concrete and reinforcement

In reality, tensile stresses in reinforced concrete do not drop immediately after exceeding the tensile strength, but forces can still be transmitted across the cracks. To consider the effect of tension stiffening in nonlinear analysis, there are two different methods. In numerical simulations tension stiffening is usually considered via a stress-strain relationship or a stress-crack opening relationship with a falling range after exceeding the tensile strength. As an alternative, the tension stiffening may be considered via a modified stress-strain relationship of the reinforcement according to [11]. The stiffness of the reinforcement is increased depending on the cracking in the concrete in order to consider the concrete's effect. In figure 3 (right) the solid line corresponds to the course from [11]. As a further simplification and if repeated loading



Modelling of the concrete has no visible influence here. Since the concrete in the composite beams essentially only acts in compression, a model that neglects the reinforcement also achieves reliable results. Deviations only occur in the elastic range due to the neglect of tension stiffening.

The parameter study on the single-span beam with concrete in compression zone clearly shows that variations in the material behaviour of the steel have a much more significant influence on the results than the material behaviour of the concrete. This agrees with the results of safety-parameter investigations on single-span composite beams from [14].

It is shown that the application of the real yield strength of the steel has the greatest influence on the accuracy of the calculation. Due to overstrength of the steel, it is therefore essential to carry out tensile tests to know the precise start of plastification. If such results are not available, a rough estimate can only be made based on the strength classes. It is evident that the steel can also be approximated bilinearly according to EN 1993-1-5 [13]. In addition, it has been shown in figure 4 that the concrete can also be bilinearly approximated and the reinforcement can be neglected.

## 5 VERIFICATION OF THE MODEL

### 5.1 Introduction

To verify the analysis model described above, comparative calculations with existing tests on single-span composite beams were carried out. Furthermore, the application of the framework models to composite beams with negative moments, i.e., with concrete in the tension zone, is to be checked. For this purpose, comparative calculations are carried out on symmetrical double-span beams.

In addition, scientific numerical simulations were carried out for the investigated single-span beams using the finite elements programme Abaqus CAE [15]. The model consists out of a three-dimensional model in which the mesh was created using very detailed volume elements. In the composite joint, each individual composite element was modelled three-dimensionally and connected to the surrounding concrete by defining contact surfaces. The concrete damaged plasticity model implemented in Abaqus was used for modelling the concrete. The multi-axial stress state of the concrete is considered via the yield surface according to Drucker-Prager. The damage of the concrete under compressive and tensile stress is modelled via the reduction of the stiffness of individual elements using damage parameters.

### 5.2 Single-span composite beams

The following three single-span composite beam tests from [8] are identical in terms of material, structural system and cross-section, differing only in the partial shear connection ratio of about 100%, 70% and 40%. The composite beams consist of a steel profile IPE 400 out of S235 and a concrete slab out of C30/37 ( $w \times h$  1500 mm x 140 mm) which are connected with headed studs (diameter 16 mm, height 100 mm). The beams span 7.70 m and are symmetrically loaded by two concentrated loads with a distance of 1.60 m. Figure 5 shows the load-deflection curves of the three composite beams in addition to the previously described beam properties. For each beam, the test results (blue), the results of the numerical simulations (grey) and the results of the framework models (red) are shown. The red solid lines represent the framework model calculations using the material behaviour from figure 3, the dashed lines the simplified material assumptions.

In general, a close accuracy of the test results with the numerical simulations as well as the modelling by framework models is evident. The loss of stiffness of the beams always occurs in a very similar range. While all results are very close to each other for the composite beams with low shear connection, the deviations of the results of the framework model and simulations from the real test results also increase with increasing shear connection. Due to the significantly higher utilisation of the steel profile, which is caused by the higher load of the test beams with higher shear action, the deviation between precise and simplified material modelling also increases. It seems, that the flexibility of the composite joint is more accurately represented in the model than the material behaviour of the beam elements. To investigate the influence of the overestimation of the concrete at compression strength, the maximum concrete

compression strains were determined from the framework models. Concrete compression strain above  $\epsilon_{c,1}$  does not occur with the beams “SSB 40%” and “SSB 70%”, thus no overestimation of the concrete occurs here. Only with beam “SSB 100%” a compression strain of  $\epsilon_{c,1}$  exceeded from a load of 520 kN and thus the load-bearing capacity of the concrete is exceeded. This effect can be seen in figure 5 above this load level.

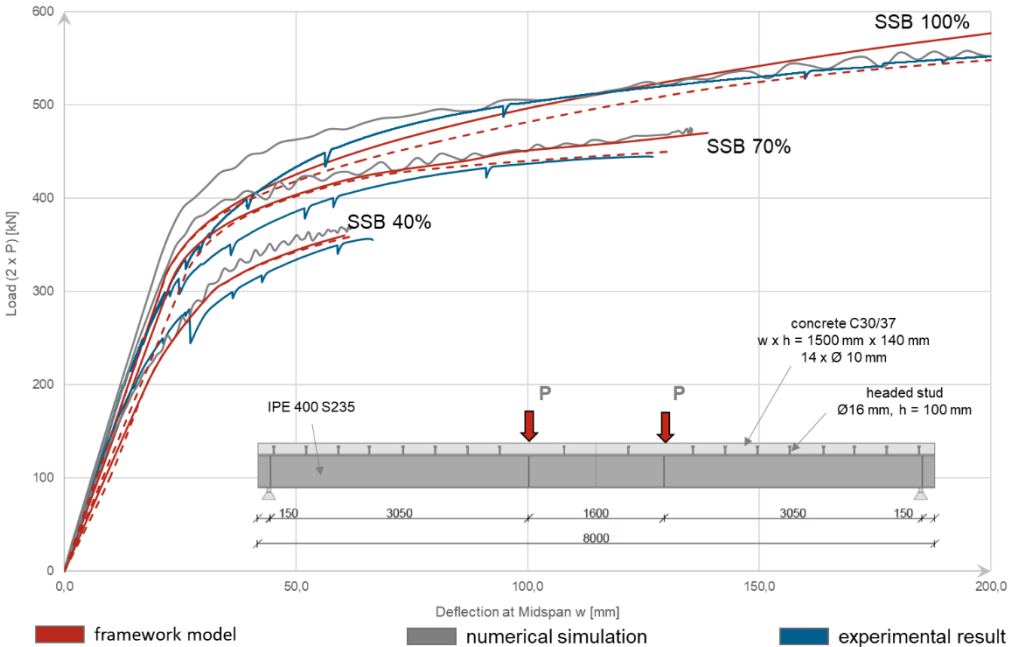


Figure 5. Load-deflection-curve of the three single-span beams

According to [8], the failure of the three test beams occurs after huge deformations due to a sudden fracture of the composite joint, in which the concrete slab separated over the entire length of one beam half. This failure is indicated in the load-end slip curves in figure 6 by a horizontal deformation curve at the end of the test. The sudden failure cannot be represented in the framework models. However, an almost horizontal course of the slip at beam end can also be seen in the results of the framework model, which is comparable to a failure of the bonded joint.

Figure 6 shows the load end slip curves of the three beams on the left. Since in the real tests, unlike in the model calculations, there was no absolute symmetry in the beams, the results from both beam-ends are shown. It can be seen that in the test, despite preloading cycles, it was not possible to eliminate the adhesive bond between the concrete and the steel. At the beginning of the tests, the load increases without slip occurring. Only at higher load levels the bond failed, and slip occurred.

While the load-slip curves of the framework models in the range of bearing capacity show very good accuracy with the real tests, clear differences occur at the lower load levels. This is due to the simplified trilinear load-slip characteristic of the framework models. Since the push-out tests succeeded better in loosening the adhesive bond through preloading cycles, this also results in a deviation between the dowel characteristic curve and the real component behaviour. In general, the load-slip behaviour can be represented well by means of a framework model; with more precise input values, the test results would be even more accurate. On the right in Figure 6 the slip curves of the three beams are shown over the beam

length. It is shown that the curves of the framework models (red) correspond very precisely to the individual measuring points of the beam tests (blue dots).

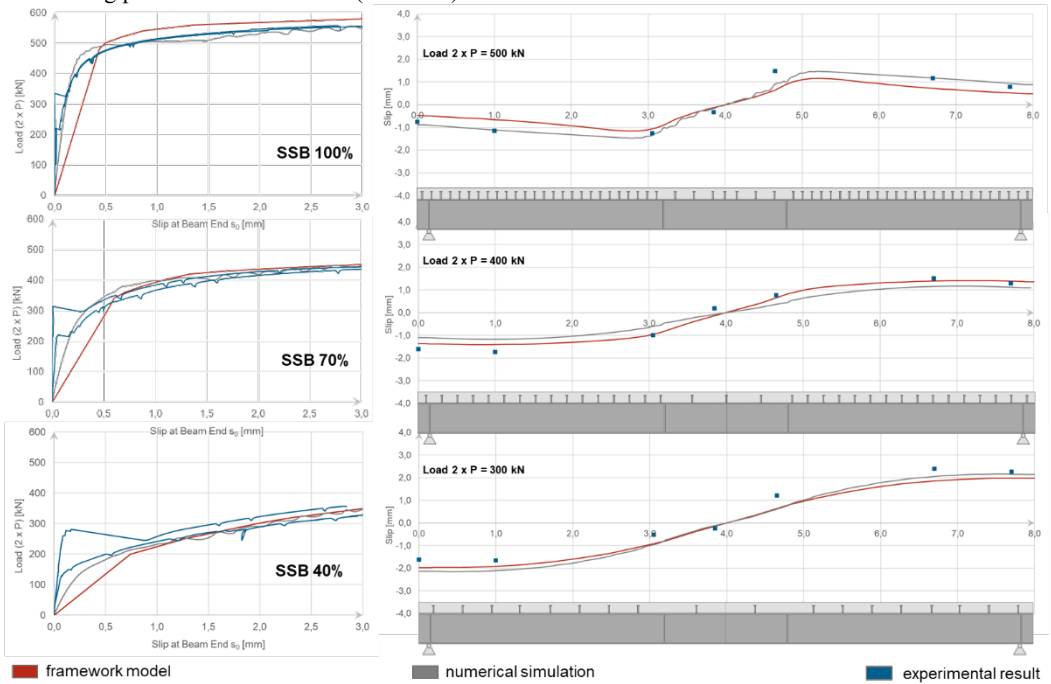


Figure 6. left: Load-slip relationship; right slip along the beam

Figure 7 illustrates the uplift of the slab of “SSB 70%” in detail. On the left, the load uplift curve at beam-end is shown. It is evident that there is good accuracy between the test and the simulations although there is a less stiff behaviour in the plastic range of the simulation. The simulations have shown a difference between local and global uplift effects. Whereas locally at the headed stud the uplift decreases to zero due to the compression, globally, i.e. between the edge of the steel profile and the concrete slab, uplift occurs almost over the entire length of the beam. Only the global uplift is presented in the following.

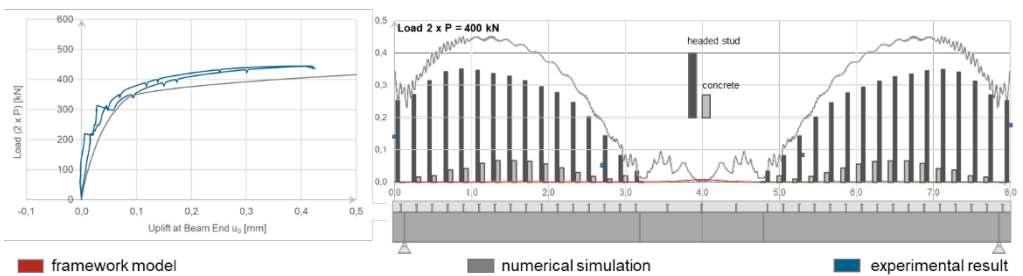


Figure 7. Uplift of the slab of “SSB 70%”

On the right, the grey curve shows the course of the uplift along the beam length at the load level of 400 kN as a result of the simulation. The four blue points represent measurements from the test. In red the course the uplift from the framework model with horizontal spring arrangement is shown. Even though

slight uplift occurs, it is clear that it is of a much smaller order of magnitude than in the real tests. It is obvious that due to the modelling and the resulting deformation of the springs, a realistic modelling of the uplift effects was not possible. As mentioned at the beginning, an inclined arrangement of the "horizontal" spring could generate an uplift initiated by the slip as it occurs in the real construction, however here the uplift would be predetermined in the model.

Below the uplift curve, the magnitude of the longitudinal deformation of the headed stud as well as the magnitude of the concrete compression, which in total lead to the uplift, are shown by a bar chart. It is visible that at each point the uplift results mainly from the elongation of the headed stud. The assumption of the axial stiffness of the bolt shaft is therefore justified for the present composite joint. It is visible from the load-uplift curve that a non-linear behaviour of the bonded joint also occurs in vertical direction. The vertical stiffness would therefore have to be at least a multi-linear curve considering the plasticising of the headed stud.

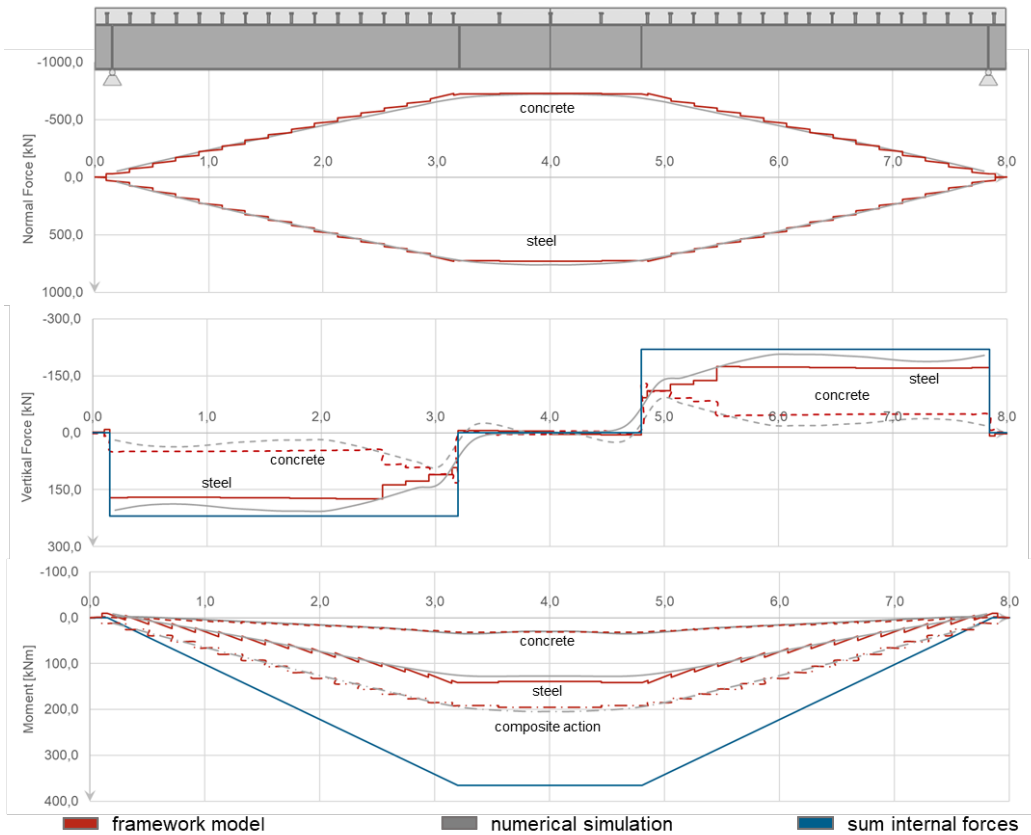


Figure 8. Internal forces of the beam "SSB 70%"

Figure 8 shows an example of the internal forces of the beam "SSB 70%" at the load level  $2 \times P = 440 \text{ kN}$ , i.e., a load level far in the non-linear range of the load-deformation curve. The results of the partial internal forces of concrete and steel determined by means of a framework model and numerical simulations show very good consistency. Due to the framework model and the transmission of the forces at the defined nodes, the result curves show step changes at these nodes. While the curves of normal forces and moments show very good accuracy, where even the plastification of the steel is well represented, deviations occur in

the distribution of shear forces. In general, it is visible that with the simplified modelling using framework models, a realistic distribution of the internal forces to the partial cross-sections is achieved and thus a realistic load-bearing behaviour of the model is assumed.

### 5.3 Double-span composite beams

In addition to the single-span composite beams, the framework models are also applied to multi-span composite beams or other beams with a concrete slab in tension. Here, comparative analyse is made for a 14 m long symmetrical composite beam with two spans with a partial shear connection ratio of about 40%. In the cross-sections, the beam has the similar properties as the previously investigated single span beams, except for different reinforcement. The exact properties are shown in figure 9. As the tests were just carried out, the results of the framework models are only compared with the experimental results of the beam, not with numerical simulations.

Figure 9 shows the load-deflection curve. It is to be seen that the non-linear deformation behaviour of two-span beams, where the modelling of the concrete in the tensile area plays an essential role, can also be reproduced effectively with the framework models presented here. The deviations in the transfer from elastic to plastic material behaviour are of the same order of magnitude as for the single span beams.

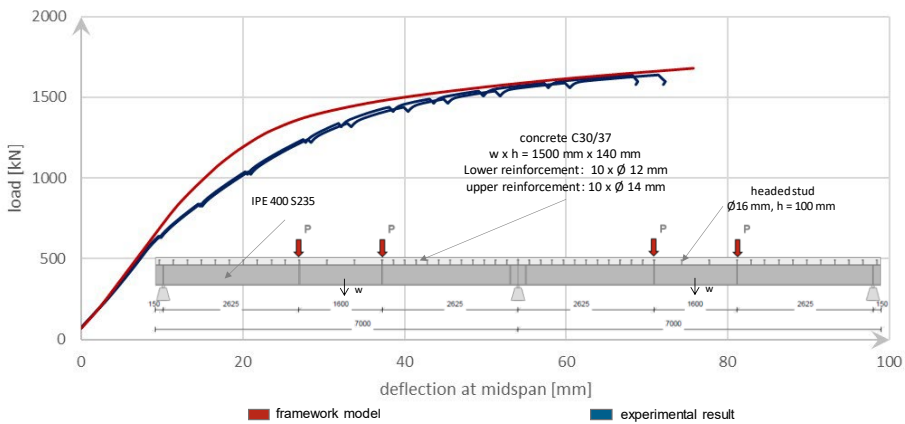


Figure 9. Load-deflection-curve of the double-span beam

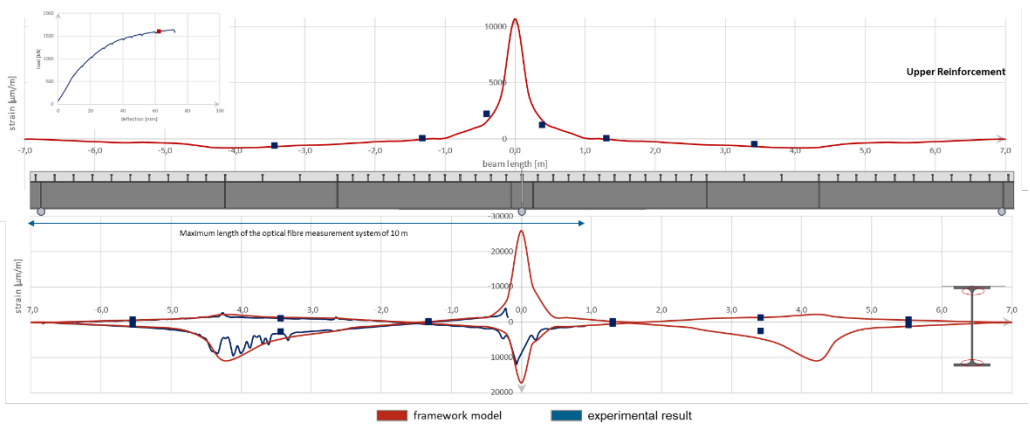


Figure 10. Strains along the beam; top: upper reinforcement; bottom: upper and lower chord of the steel profile

Figure 10 presents a comparison of the framework model's steel strains with the results of the experiment at a load of 1600 kN. The upper course shows the results of the upper reinforcement and the lower courses represents the strains at steel profile measured at the upper and lower chords. The blue squares represent the measured results of the strain gauges and the two blue curves represents the results of the fibre optical measurement system measured continuously over the length at the steel profile. Due to the maximum length of this system, the strains could not be measured at the whole length. Here, as well for the steel profile as for the reinforcement a high accuracy of the framework model even for the non-linear material behaviour could be seen.

Figure 11 shows the load-slip-relations at the marked sections. Each red ellipsis represents the position of a displacement transducer for slip measurement in the beam test. Due to the use of symmetry in the diagram, at each measuring point two experimental results are shown. It is visible that the result of the framework model corresponds very accurately with the test results at every measurement point.

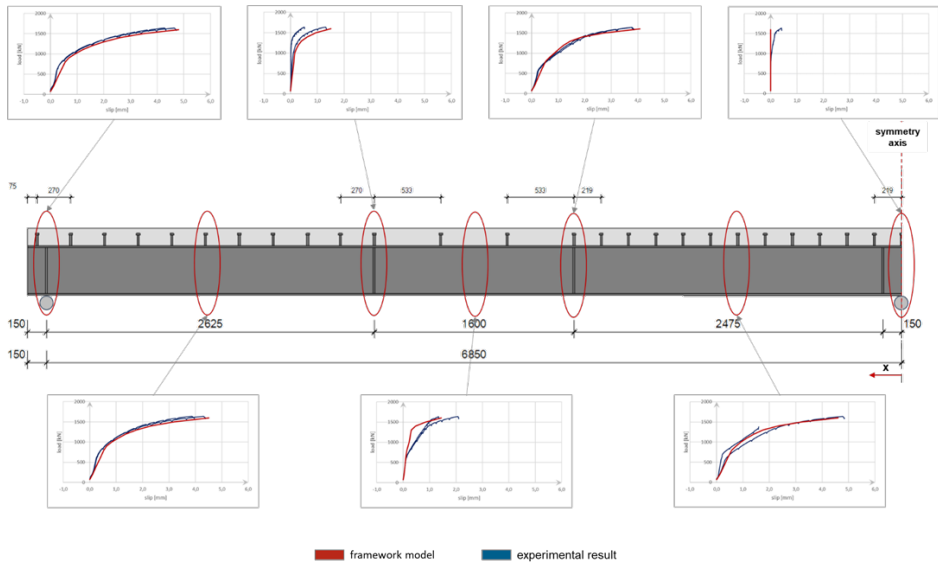


Figure 11. Load-slip relationship of the double-span beam at various sections

## 6 CONCLUSIONS

Based on the presented comparisons of the framework models with the test results of the beam tests of various static systems and partial shear connection ratios, it is demonstrated that the load bearing and deformation behaviour of composite beams can be modelled with good accuracy by framework models considering the non-linear material behaviour as well as the non-linear behaviour of the composite joint. While there are slight deviations in the load-deflection curves, the slip curves could also be reproduced very well, especially its non-linear behaviour. In addition, the good consistency of the curvatures and the internal force distribution shows clearly that a realistic load-bearing behaviour is modelled. Further investigations are necessary to determine the uplift behaviour, especially regarding the determination of the non-linear load-uplift curve as vertical spring stiffness as well as the arrangement of the spring in the model. Due to the high level of correlation between model calculations and real tests, it is possible to use the beam models in research as a complement to complex numerical simulations.

Since the real beam tests could be represented with high consistency, the framework model seems to be generally suitable for non-linear, strain-limited designs. While the non-linear material properties of steel, concrete and reinforcement could be derived from the respective standards, improved information would have to be developed for the representation of the composite joint depending on the type of shear connector as well as the concrete strength.

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