

# Quantisation of 2D-gravity with Weyl and area-preserving diffeomorphism invariances

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## Abstract

The constraint structure of the induced 2D-gravity with the Weyl and area-preserving diffeomorphism invariances is analysed in the ADM formulation. It is found that when the area-preserving diffeomorphism constraints are kept, the usual conformal gauge does not exist, whereas there is the possibility to choose the so-called “quasi-light-cone” gauge, in which besides the area-preserving diffeomorphism invariance, the reduced Lagrangian also possesses the  $SL(2, \mathbb{R})$  residual symmetry. This observation indicates that the claimed correspondence between the  $SL(2, \mathbb{R})$  residual symmetry and the area-preserving diffeomorphism invariance in both regularisation approaches does not hold. The string-like approach is then applied to quantise this model, but a fictitious non-zero central charge in the Virasoro algebra appears. When a set of gauge-independent  $SL(2, \mathbb{R})$  current-like fields is introduced instead of the string-like variables, a consistent quantum theory is obtained, which means that the area-preserving diffeomorphism invariance can be maintained at the quantum level.

The induced 2D-gravity with Weyl and area-preserving diffeomorphism invariances has attracted much attention recently [1-7]. As is well known, in the conventional regularisation approach to 2D gravity [8-11], the diffeomorphism invariance is preserved, while the Weyl invariance is lost. In the path integral formulation, this can be accomplished by choosing the diffeomorphism-invariant, but not Weyl-invariant measures for the functional integrations [10,11]. Nevertheless, one can adopt an alternative regularization approach in which part of the diffeomorphism invariance is sacrificed so as to obtain a Weyl-invariant theory. This alternative approach is motivated by the observation that the Lagrangian, invariant classically with respect to reparametrization and Weyl transformation, depends on the metric only through the Weyl-invariant combination [1-3]. From the conformal geometry's point of view, this idea is based on the fact that the amount of gauge degrees of freedom provided by the diffeomorphism group  $Diff\mathcal{M}$  of manifold  $\mathcal{M}$  is equivalent to the amount provided by the combination of the Weyl rescaling and the area-preserving diffeomorphism subgroup  $SDiff\mathcal{M}$  of  $Diff\mathcal{M}$  [7]. Recently, the anomalous Ward identities and part of the constraint structure for the induced 2D-gravity with Weyl and area-preserving diffeomorphism invariances have been discussed in Ref.[3]. Especially, some physical results associated with area-preserving diffeomorphism invariance, like 2D Hawking radiation [12], have been studied in Refs.[4,5]. However, there are still some questions that need to be answered. For example, in the induced 2D-gravity with reparametrization invariance, when the diffeomorphism constraints are kept, i.e., no gauge fixings are chosen for them, one has the freedom to choose the conformal or light-cone gauge. A natural question arises, in the induced 2D-gravity with area-preserving diffeomorphism invariance, when the Virasoro generators are required to annihilate the physical states, whether there is the possibility to pick up the above two gauges. On the other hand, the effective Lagrangian in the present case has Weyl and area-preserving diffeomorphism invariance, so that one may ask whether the latter invariance is broken or not at the quantum level, and supposing it is lost, whether there exists a consistent approach to quantise this theory, that is, whether the area-preserving diffeomorphism invariance can be maintained at the

quantum level.

In the present paper, the ADM formulation [13-16] is applied to analyse the constraint structure for the induced 2D-gravity with the Weyl and area-preserving diffeomorphism invariances. It is shown that when the area-preserving diffeomorphism constraints are kept, the usual conformal gauge does not exist, whereas there is the possibility to choose the so-called “quasi-light-cone” gauge. It is first found that in the “quasi-light-cone” gauge, there is an  $SL(2,R)$  residual symmetry in the reduced Lagrangian. Even though the  $SL(2,R)$  currents manifest themselves as generators of the residual symmetry in the “quasi-light-cone” gauge, we find that these currents can be defined in a gauge-independent way. As indicated in Ref.[1], the Weyl-invariant approach has some resemblance with Polyakov’s light-cone approach [9] where there is an  $SL(2,R)$  residual symmetry which was thought to be analogous to the area-preserving group in the Weyl-invariant approach. However, it is found that in the Weyl-invariant regularisation approach, besides the area-preserving diffeomorphism invariance, the reduced Lagrangian also possesses the  $SL(2,R)$  residual symmetry in the “quasi-light-cone” gauge. In this sense, the correspondence between the  $SL(2,R)$  residual symmetry and the area-preserving diffeomorphism invariance in both regularization approaches does not exist. To quantise this theory, the string-like approach is exploited, but a non-zero central charge in the Virasoro algebra appears, and so the resulting quantum theory of gravity is anomalous, that is, at the quantum level the area-preserving diffeomorphism invariance is broken. However, this anomaly is only fictitious in the sense that it can be avoided. In fact, in order to see that a consistent quantisation of this theory is possible, a set of current-like fields is introduced instead of the string-like variables, and the consistency condition is the vanishing of the total central charge, which establishes the relation between the constant  $K$  and the matter central charge  $C_m$ . Therefore, a consistent quantum theory is obtained, which means that the area-preserving diffeomorphism invariance can be maintained at the quantum level.

We start from the effective action with Weyl and area-preserving diffeomorphism

invariance [1-7]:

$$S = \int d^2x \left[ -\frac{1}{2} \gamma^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + \alpha R(r) \phi \right] \quad (1)$$

where

$$\gamma^{\mu\nu} \equiv \sqrt{-g} g^{\mu\nu}, \quad (2)$$

is Weyl invariant and  $\phi$  is an auxiliary field, and for simplicity in the following discussion  $\alpha = 1$  is chosen. In fact, one may set the  $R\phi$  coefficient to be an arbitrary number  $n$  [6]. Following the ADM formulation, the metric can be parametrized as [13-16]:

$$g_{\alpha\beta} = e^{2\rho} \begin{pmatrix} -\sigma^2 + \theta^2 & \theta \\ \theta & 1 \end{pmatrix} \quad (3)$$

where  $\sigma(x)$  and  $\theta(x)$  are lapse and shift functions respectively, and the conformal factor  $e^{2\rho}$  has been factored out. In terms of this parametrization, the action (1) can be written as

$$S = \int d^2x \left[ \frac{\dot{\phi}^2}{2\sigma} - \frac{\theta}{\sigma} \dot{\phi} \phi' - \frac{\sigma^2 - \theta^2}{2\sigma} \phi'^2 + \frac{\dot{\phi}}{\sigma} \left( 2\theta' - \frac{\theta\sigma'}{\sigma} + \frac{\dot{\sigma}}{\sigma} \right) + \frac{\phi'}{\sigma} \left( \sigma\sigma' - 2\theta\theta' - \frac{\theta\dot{\sigma}}{\sigma} + \frac{\theta^2\sigma'}{\sigma} \right) \right] \quad (4)$$

which is independent of the conformal factor  $\rho$ , and so is Weyl-invariant. In the above, dots and primes denote differentiation with respect to time and space respectively. The canonical momenta associated with the fields  $\{\theta, \sigma, \phi\}$  are:

$$\pi_\theta \approx 0 \quad (5)$$

$$\pi_\sigma = \frac{\dot{\phi}}{\sigma^2} - \frac{\theta\phi'}{\sigma^2}$$

$$\pi_\phi = \frac{\dot{\phi}}{\sigma} - \frac{\theta\phi'}{\sigma} + \frac{1}{\sigma} \left( 2\theta' - \frac{\theta\sigma'}{\sigma} + \frac{\dot{\sigma}}{\sigma} \right) \quad (6)$$

with Poisson brackets

$$\{\theta(x), \pi_\theta(y)\} = \{\sigma(x), \pi_\sigma(y)\} = \{\phi(x), \pi_\phi(y)\} = \delta(x - y) \quad (7)$$

As is usually done, Eq.(5) has to be taken as a primary constraint and it holds only in a “weak” sense. The canonical Hamiltonian, up to surface terms, is

$$H_c = \int dx \left[ \theta(3\sigma'\pi_\sigma + 2\sigma\pi'_\sigma + \pi_\phi\phi') + \sigma^2\pi_\sigma\pi_\phi - \frac{1}{2}\sigma^3\pi_\sigma^2 + \frac{1}{2}\sigma(\phi')^2 - \sigma'\phi' \right] \quad (8)$$

The consistency of the constraint  $\pi_\theta \approx 0$  under time evolution requires that

$$G_\theta(x) = -\{\pi_\theta(x), H_c\} = 3\sigma'\pi_\sigma + 2\sigma\pi'_\sigma + \pi_\phi\phi' \approx 0 \quad (9)$$

which is a secondary constraint, and satisfies

$$\{G_\theta(x), G_\theta(y)\} = [G_\theta(x) + G_\theta(y)]\partial_x\delta(x-y)$$

According to the Dirac Hamiltonian procedure for constrained systems, the Poisson brackets of the Hamiltonian with the constraints should vanish weakly, so we have

$$\{H_c, G_\theta(x)\} \approx [\sigma G_\sigma(x)]' \approx 0 \quad (10)$$

with

$$G_\sigma(x) = \sigma\pi_\sigma\pi_\phi - \frac{1}{2}\sigma^2\pi_\sigma^2 + \frac{1}{2}\phi'^2 - \frac{\sigma'\phi'}{\sigma} + \frac{2(\phi'\sigma)'}{\sigma} - \frac{c}{\sigma} \approx 0 \quad (11)$$

where  $c$  is an arbitrary constant, and the canonical Hamiltonian, up to surface terms, turns out to be:

$$H_c = \int dx(\theta G_\theta + \sigma G_\sigma + c)$$

From Eq.(7), after a series of steps, we have

$$\begin{aligned} \{G_\sigma(x), G_\sigma(y)\} &= [G_\theta(x) + G_\theta(y)]\partial_x\delta(x-y) \\ \{G_\theta(x), G_\sigma(y)\} &= [G_\sigma(x) + G_\sigma(y)]\partial_x\delta(x-y) \\ \{G_\theta(x), G_\theta(y)\} &= [G_\theta(x) + G_\theta(y)]\partial_x\delta(x-y) \end{aligned} \quad (12)$$

Eq.(12) shows that  $G_\sigma, G_\theta$  form the diffeomorphism algebra under the Poisson bracket operation, that is, they are first-class constraints at the classical level. However, one can see that  $G_\sigma, G_\theta$  generate the area-preserving diffeomorphism transformation, not the general diffeomorphism transformation. It can be seen from Eq.(9), i.e.

$$\delta\phi(x) = -\left\{\int \epsilon(y)G_\theta(y)dy, \phi(x)\right\} = \epsilon(x)\partial_x\phi(x),$$

that under the infinitesimal transformation, the field  $\phi$  behaves as a scalar field, but from Eq.(1), we find that  $\phi$  is not a scalar field under the reparametrization transformation [2].

As claimed in Ref.[7], the field  $\phi$  is a scalar field only with respect to the area-preserving diffeomorphism transformation. So we conclude that  $G_\sigma, G_\theta$  are the generators of the area-preserving diffeomorphism symmetry.

When the area-preserving diffeomorphism invariance is kept, i.e., no gauge fixings are chosen for  $G_\sigma, G_\theta$ , we can just choose a gauge fixing for the constraint  $\pi_\theta \approx 0$ , then we find the usual conformal gauge does not exist. However, there is the so-called “quasi-light-cone” gauge in which the reduced Lagrangian possesses the  $SL(2, R)$  residual symmetry. To discuss the “quasi-light-cone” gauge, we need a field redefinition which is given by

$$\sigma = (B + 1)^{-1} \quad (13)$$

then the “quasi-light-cone” can be defined by choosing the gauge fixing for the constraint  $\pi_\theta \approx 0$  as

$$\theta = B(B + 1)^{-1} \quad (14)$$

i.e., the invariant line element is

$$ds^2 = -\frac{2e^{2\rho}}{B + 1} [dx^+ dx^- - B(dx^+)^2]$$

From Eqs.(13,14), the action (4) is reduced to

$$S = \int d^2x [\partial_+ \phi \partial_- \phi + B(\partial_- \phi)^2 - 2\partial_- B \partial_- \phi] \quad (15)$$

where  $x^\pm = (\frac{1}{\sqrt{2}})(x^0 \pm x^1)$ ,  $\partial_\pm = (\frac{1}{\sqrt{2}})(\partial_0 \pm \partial_1)$ . It can be verified that the action (15) is invariant under the residual symmetry transformations

$$\begin{aligned} \delta\phi &= \epsilon \partial_- \phi - \partial_- \epsilon \\ \delta B &= -\partial_+ \epsilon + \epsilon \partial_- B - B \partial_- \epsilon \\ \partial_-^3 \epsilon &= 0 \end{aligned} \quad (16)$$

From action (15), the momenta conjugate to  $\phi$  and  $B$  are

$$\begin{aligned} \pi_\phi &= (1 + B)(\dot{\phi} - \phi') - \phi' - \dot{B} + B' \\ \pi_B &= -\dot{\phi} + \phi' \end{aligned} \quad (17)$$

The transformation (16) is generated by the charge  $Q = \int dx q(x)$  with

$$q(x) = \epsilon \left[ -\frac{B+1}{\sqrt{2}} T_{--} + \sqrt{2} \partial_-^2 B \right] - \sqrt{2} (\partial_- \epsilon) (\partial_- B) + \sqrt{2} (\partial_-^2 \epsilon) B \quad (18)$$

where  $T_{--}$  is the  $(--)$  component of the energy-momentum tensor given by

$$T_{--} = (\partial_- \phi)^2 + 2 \partial_-^2 \phi$$

If we make a decomposition of  $\epsilon(x)$  with

$$\epsilon = \frac{1}{\sqrt{2}} [\epsilon^-(x^+) + 2x^- \epsilon^0(x^+) + (x^-)^2 \epsilon^+(x^+)] \quad (19)$$

the generator  $q(x)$  becomes

$$q(x) = \frac{1}{2} \epsilon^-(x^+) j^+ - \epsilon^0(x^+) j^0 + \frac{1}{2} \epsilon^+(x^+) j^- \quad (20)$$

with

$$\begin{aligned} j^+ &= -(1+B) T_{--} + 2 \partial_-^2 B \\ j^0 &= \sqrt{2} \partial_- B - x^- j^+ \\ j^- &= 4B - 2x^- j^0 - (x^-)^2 j^+ \end{aligned} \quad (21)$$

Using the canonical commutation relations of  $\phi$  and  $B$ , the currents (21) obey the  $SL(2, \mathbb{R})$  algebra

$$\{j^a(x), j^b(y)\} = -2\sqrt{2} \epsilon^{abc} \eta_{cd} j^d(x) \delta(x-y) + 4\eta^{ab} \delta'(x-y) \quad (22)$$

with

$$\eta^{ab} = \begin{pmatrix} 0 & 0 & 2 \\ 0 & -1 & 0 \\ 2 & 0 & 0 \end{pmatrix}, \quad \epsilon^{-0+} = 1$$

where  $a, b, c = -, 0, +$ . Eqs.(12,15,16,20,21) show that in the ‘‘quasi-light-cone’’ gauge, besides the area-preserving diffeomorphism invariance, there is a  $SL(2, \mathbb{R})$  residual symmetry. Then we conclude that the  $SL(2, \mathbb{R})$  residual symmetry in Polyakov’s light-cone approach [9] does not correspond to the area-preserving diffeomorphism symmetry in the Weyl-invariant approach [1-7].

From the above discussion, we see that the  $SL(2,R)$  currents manifest themselves as generators of the residual symmetry in the “quasi-light-cone” gauge, but these currents can be defined in a gauge-independent way by the following expressions:

$$\begin{aligned}
j^+ &= \sigma(G_\sigma - G_\theta) \\
&= \sigma^2\pi_\sigma\pi_\phi - \frac{1}{2}\sigma^3\pi_\sigma^2 + \frac{1}{2}\sigma\phi'^2 + \sigma'\phi' + 2\sigma\phi'' - 3\sigma\sigma'\pi_\sigma - 2\sigma^2\pi'_\sigma - \sigma\pi_\phi\phi' \\
j^0 &= \sqrt{2}\left(-\sigma\pi_\sigma + \pi_\phi + \frac{\sigma'}{\sigma}\right) - x^-j^+ \\
j^- &= 4\left(\frac{1}{\sigma} - 1\right) - 2x^-j^0 - (x^-)^2j^+
\end{aligned} \tag{23}$$

For simplicity, the arbitrary constant  $c$  in  $G_\sigma$  has been chosen to be zero from now on. One can easily check that the currents defined above satisfy Eq.(22), and Eq.(23) can be reduced to Eq.(21) in the “quasi-light-cone” gauge.

From Eq.(23) one can obtain

$$\sigma = \left\{ 1 + \frac{1}{4} \left[ j^- + 2x^-j^0 + (x^-)^2j^+ \right] \right\}^{-1} \tag{24}$$

which shows that the graviton field can be expressed in terms of the  $j$  variables.

In order to quantise this theory by the string-like approach, we perform the canonical change of the original variables by

$$\psi = \phi + \ln \sigma, \quad \eta = -\ln \sigma$$

and

$$\pi_\psi = \pi_\phi, \quad \pi_\eta = \pi_\phi - \pi_\sigma e^{-\eta} \tag{25}$$

In terms of the new canonical variables defined in (25), the area-preserving diffeomorphism constraints can be written as

$$\begin{aligned}
G_\sigma &= \frac{1}{2}\psi'^2 + \frac{1}{2}\pi_\psi^2 + 2\psi'' - \frac{1}{2}\eta'^2 - \frac{1}{2}\pi_\eta^2 + 2\eta'' \\
G_\theta &= \pi_\psi\psi' + 2\pi'_\psi + \pi_\eta\eta' - 2\pi'_\eta
\end{aligned} \tag{26}$$

which are equivalent to the constraints

$$G_\pm = \frac{1}{2}(G_\sigma \pm G_\theta) \tag{27}$$



which obey the Virasoro algebra

$$\begin{aligned}\{G_{\pm}(x), G_{\pm}(y)\} &= \mp[G_{\pm}(x) + G_{\pm}(y)]\partial_x\delta(x-y) \\ \{G_+(x), G_-(y)\} &= 0\end{aligned}$$

When we take the Fourier transform of the constraint  $G_+$

$$L_n = \int_{-\pi}^{\pi} dx \exp(inx) G_+(x)$$

we have

$$\{L_n, L_m\} = i(n-m)L_{n+m} \quad (28)$$

If we carry on the analogy with string theory by defining the oscillator variables

$$\begin{aligned}a_n &= \int_{-\pi}^{\pi} dx \exp(inx)(\pi\psi + \psi') \\ b_n &= \int_{-\pi}^{\pi} dx \exp(inx)(\pi\eta - \eta')\end{aligned} \quad (29)$$

which satisfy

$$\{a_n, a_m\} = -\{b_n, b_m\} = -2in\delta_{n,-m} \quad (30)$$

then the Virasoro operators have the familiar form

$$L_n = \frac{1}{2} \sum_m a_{n-m} a_m - \frac{1}{2} \sum_m b_{n-m} b_m - in(a_n - b_n)$$

Considering the induced 2D-gravity with Weyl and area-preserving diffeomorphism invariance described by the two sets of operators  $a_n$  and  $b_n$  with commutation relations taken from (30), we can define the quantum Virasoro generators using normal ordering

$$L_n = \frac{1}{2} \sum_m : a_{n-m} a_m : - \frac{1}{2} \sum_m : b_{n-m} b_m : - in(a_n - b_n) \quad (31)$$

and the vacuum state  $|0\rangle$  is defined by

$$L_n|0\rangle = 0, \quad n > 0, \quad L_0|0\rangle = a_0|0\rangle \quad (32)$$

If such a prescription is followed, a non-zero central charge in the Virasoro algebra appears [17-19], and the resulting quantum theory of gravity is anomalous.

However, if the string-like variables  $\psi$  and  $\eta$  are replaced by a set of current-like fields, a consistent quantum theory can be achieved. First we introduce a new field  $h$  given by

$$h = \pi_\phi + \phi' + \frac{\sigma'}{\sigma} \quad (33)$$

with

$$\begin{aligned} \{h(x), h(y)\} &= 2\partial_x \delta(x-y) \\ \{h(x), j^a(y)\} &= 0, a = -, 0, + \end{aligned}$$

Eq.(24) shows that the field  $\sigma$  can be expressed in terms of the  $j$  variables, while  $\pi_\sigma, \pi_\phi, \phi$  can be solved using (23) and (33) with appropriate boundary conditions. Since the set of variables  $(j^-, j^0, j^+, h)$  is defined in a gauge-independent way, we then describe the whole theory in terms of them, and the constraints can be expressed as

$$\begin{aligned} G_\theta &= G_m + G_s - \frac{x^-}{\sqrt{2}} j^{+'} \approx 0 \\ G_\sigma &= G_\theta + \frac{j^+}{\sigma} \approx 0 \end{aligned}$$

with

$$\begin{aligned} G_m &= \frac{1}{4} h^2 + h' \\ G_s &= \frac{1}{8} \eta_{ab} j^a j^b - \frac{1}{\sqrt{2}} j^{0'} \end{aligned} \quad (34)$$

From (34), we find the old constraints  $(G_\theta, G_\sigma)$  are equivalent to

$$\begin{aligned} j^+ &\approx 0 \\ b &= G_m + G_s \approx 0 \end{aligned} \quad (35)$$

which obey the algebra

$$\begin{aligned} \{j^+(x), j^+(y)\} &= 0 \\ \{b(x), j^+(y)\} &= -[\partial_x j^+(x)] \delta(x-y) \\ \{b(x), b(y)\} &= [b(x) + b(y)] \partial_x \delta(x-y) \end{aligned} \quad (36)$$

Consider the algebra of  $G_m$  (where the  $h$  field enters), we have

$$\begin{aligned}\{G_m(x), G_m(y)\} &= [G_m(x) + G_m(y)]\delta'(x-y) - 2\delta'''(x-y) \\ \{G_m(x), G_s(y)\} &= 0\end{aligned}\tag{37}$$

which shows that  $G_m$  retains some memory of the matter fields through the semi-classical central charge in (37). Therefore we can consider the  $h$  field as carrying the matter central charge in this representation of the theory [17]. According to the light-cone gauge prescription [9], we assume the  $j^a$  operators satisfy equal-time commutation relations as in (22),

$$[j^a(x), j^b(y)] = -2i\sqrt{2}\epsilon^{abc}\eta_{cd}j^d\delta(x-y) + 4i\bar{\alpha}^2\eta^{ab}\delta'(x-y)\tag{38}$$

allowing a possible renormalization of the constant  $\alpha$  in action (1), and the renormalized constant  $\bar{\alpha}$  is  $K/384$ , where  $K$  is to be determined by the consistency condition. Then the quantum constraint operators can be defined as

$$\begin{aligned}j^+ &\approx 0 \\ b &=: G_m : + : G_g : + : G_s : \approx 0\end{aligned}\tag{39}$$

where

$$: G_s := \frac{48\pi}{K+2} : \eta_{ab}j^aj^b : - \frac{1}{\sqrt{2}}j^{0'} \approx 0$$

is the renormalized gravitational energy-momentum contribution and  $G_g$  is the ghost piece coming from the gauge fixing. The usual physical state can be defined as

$$j^+|physical\rangle = 0, \quad b|physical\rangle = 0\tag{40}$$

Eq.(40) demands that the algebra of the constraints  $(j^+, b)$  has no Schwinger terms. Then the consistency condition is the vanishing of the total central charge of  $[b, b]$  [17]

$$C_T = C_m + C_g + C_s = 0\tag{41}$$

with

$$C_s = 3K/(K+2) - 6K\tag{42}$$

which establishes the relation between  $K$  and the matter central charge  $C_m$ . Then a consistent quantum theory can be obtained, that is, at the quantum level the area-preserving diffeomorphism invariance has been maintained.

In conclusion, the constraint structure for the induced 2D-gravity with the Weyl and area-preserving diffeomorphism invariance has been analysed completely in the ADM formulation. It has been shown that if the area-preserving diffeomorphism constraints are kept, the usual conformal gauge does not exist, whereas we can choose the “quasi-light-cone” gauge. It has been first found that in the “quasi-light-cone” gauge, besides the area-preserving diffeomorphism invariance, there is also a  $SL(2,R)$  residual symmetry in the reduced Lagrangian. In this sense, the correspondence between the  $SL(2,R)$  residual symmetry and the area-preserving diffeomorphism invariance in both regularisation approaches does not hold. Although the  $SL(2,R)$  currents manifest themselves as generators of the residual symmetry in the “quasi-light-cone” gauge, we have found that these currents can be defined in a gauge-independent way. When the string-like approach is applied to quantise this theory, a non-zero central charge in the Virasoro algebra appears, and the resulting quantum theory of gravity is anomalous. In order to consistently quantize this theory, a set of the  $SL(2,R)$  current-like fields have been introduced instead of the string-like variables, and the consistency condition is the vanishing of the total central charge. Then a consistent quantum theory is obtained, which means that the area-preserving diffeomorphism invariance can be maintained at the quantum level.

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