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ORIGINAL RESEARCH PAPER

Optimal portfolios with sustainable assets: aspects for life insurers

Ralf Korn[1,2] · **Ajla Nurkanovic**[1]

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Abstract
Since August 2022 customers have to be asked if they are interested in sustainable investment when entering a pension contract. Hence, the provider has to be prepared to offer suitable investment opportunities. Further, the provider has to manage the new risks and chances of those assets in the whole portfolio. We therefore especially look at possible consequences for optimal portfolio decisions of a life insurer and suggest modeling approaches for the evolution of the demand and the sustainability ratings for sustainable assets. We will solve various portfolio problems under sustainability constraints explicitly and suggest further research topics. As a special feature for a life insurer, we particularly look at the role of the actuarial reserve fund and the annual declaration of its return.

Keywords Sustainable investment · Constrained portfolio problems · Sustainability ratings · Rating risk

1 Introduction

In Germany since August 2022, during the consulting process on pension products, customers have to be asked about possible aims to invest their money in sustainable assets. Of course, this also requires that product providers can provide information on the sustainability status of their products and that they can possibly offer products that are built—at least partly—of sustainable assets.

✉ Ralf Korn
korn@mathematik.uni-kl.de

Ajla Nurkanovic
nurkanov@mathematik.uni-kl.de

1 Fachbereich Mathematik, RPTU Kaiserslautern-Landau, 67653 Kaiserslautern, Germany

2 Fraunhofer Institute ITWM, Fraunhofer Platz 1, 67663 Kaiserslautern, Germany

A conceptual basis for considering investments as sustainable is the so-called EU taxonomy. In [5], the main facts, definitions and goals of the EU taxonomy regulation are summarized (see [6] for the legal text). It explicitly states the central objectives of the EU Green Deal which are

1. climate change mitigation,
2. climate change adaptation,
3. sustainable use and protection of water and marine resources,
4. transition to a circular economy,
5. pollution prevention and control,
6. protection and restoration of biodiversity and ecosystems.

Activities/investments that contribute to at least one of these objectives and do not substantially harm one of the others and in addition satisfy international standards with regard to human rights and social welfare are called *taxonomy-conformal*. Thus, besides ecological issues also the other two letters of the ESG-criterion (E = environment, S = social welfare, G = governance) are required for taxonomy conformal investment.

We will in the following neither go into the topic of how to rate an investment nor how to avoid so-called green washing (i.e. disguising a conventional investment as green by a very wide interpretation of the rules of the taxonomy or other regulations), but assume that a suitable and correct rating for our investment opportunities exists. Further, we also do not consider empirical issues of the performance of sustainable assets in comparison to conventional ones. For this, we refer the reader to the recent publications by e.g. Hartzmark and Sussman [8] or Gougler and Utz [7] and the references given therein.

While one can imagine about some short term activities (such as e.g. avoiding non-necessary packaging or reducing the amount of printing), actions that have a sustainable impact on the objectives stated in the EU taxonomy often require long-term investments (such as solar or wind parks for generating electricity, developing new concepts for reducing individual traffic, etc.). This, however, is in line with investment strategies that are used in pension funds, at least with regard to their duration. Thus, insurance companies should have the potential to offer pension products with a significant sustainability component.

While single sustainable investments are possibly easy to classify as taxonomy conformal, there are problems with classifying a fund or a typical portfolio of an insurer—such as the one constituting the actuarial reserve fund (ARF)—fully as sustainable in the sense of the taxonomy. And even more, bonds offered by countries are never purely used for sustainable actions. However, it is mandatory that a substantial amount of the money raised with government bonds should be devoted to ESG-conformal or taxonomy-conformal purposes. They should therefore be assigned a sustainability rating which could be done via rating of the issuing country.

A suitable solution for this problem is the use of the *Bloomberg rating*. Based on approx. 800 criteria, it assigns ESG-scorings between 0 and 100 (where 100 is the best value) to 11,500 companies from 83 countries (see [9] for a detailed description).

To be able to assign an ESG-rating (which we will from now on use as a synonymous for *sustainability rating*) to a typical portfolio of an insurer or to its ARF, the

Bloomberg rating is extended to the ingredients of these portfolios by Heinke in [9]. As a consequence, the existence (and acceptance!) of such a sustainability rating allows the introduction of *sustainability constraints* into portfolio optimization problems. For this, note that we obtain the portfolio rating R^{π} corresponding to a portfolio vector π as a volume-weighted mean of the individual ratings R_i of the individual assets, $i = 1, \ldots, n$, i.e.

$$R^{\pi} = \sum_{i=1}^{n} \pi_i R_i .$$

For convenience, we will in the following scale the ratings R_i to [0, 1].

It is obvious that introducing a sustainability constraint of the form $R^{\pi} \geq c$ for some constant $0 < c \leq 1$ to a portfolio problem typically leads to an optimal portfolio that has a smaller level of the value function than that of the unconstrained optimal portfolio. While this at first sight makes sustainable investment less attractive, there are various aspects that point in other directions such as the natural demand for sustainable assets to ensure a higher quality of future living or the medium-term view that non-sustainable industry will be heavily taxed, that non-sustainable resources will simply run out or that sustainable investments will promise high future returns after a built-up phase.

These aspects should also be included into models for asset price dynamics. We will take care of this by introducing a new continuous-time modeling framework that in particular looks at

- the model of the price evolution of sustainable assets,
- a demand process for sustainable investments,
- and different rating approaches for sustainable investments.

In the following sections, we first lay out the mathematical setting, then consider standard continuous-time portfolio optimization problems under different sustainability constraints and settings, analyze the impacts of the sustainability constraints and suggest possible price and rating dynamics to be examined in the future.

Although some of our suggestions (such as *rebuilding the ARF*) seem to be a bit of an unconventional nature at first glance, we are totally convinced that practitioners will be able to get the messages and transform them into innovative ways of managing the ARF and/or constructing new pension products.

Even more, with the ARF and its annual declaration, we will see in Sect. 3 that a life insurer possesses a unique framework to hedge against future sustainability risk.

As one of our main intentions is to motivate new research in the area of optimal investment under sustainability constraints and its application in the life insurance sector, Sect. 4 is devoted to stating various modeling and risk management challenges. Finally, Sect. 5 summarizes our intentions and suggestions.

2 Mathematical setting and the portfolio framework

To start, let us consider a complete probability space $(\Omega, \mathcal{F}, P)$, equipped with a right-continuous filtration $\mathbb{F} = (\mathcal{F}_t, t \in [0, T])$. We further assume that the probability space is rich enough to carry an m-dimensional Brownian motion $(W(t), \mathcal{F}_t, t \in [0, T])$. This framework is the basis for modeling the uncertainty in the portfolio problems that we will consider below. Note however that we do not require that $\mathbb{F}$ is the Brownian filtration as we might also include jump processes into our framework when it comes to modeling changes in sustainability ratings in our application. Explicit requirements will be stated in the following sections.

Besides the usual ingredients of a portfolio optimization problem such as asset price dynamics $B(t), S_i(t), i = 1, \ldots, d, t \in [0, T]$ (where $B(t)$ denotes the evolution of the money market account, $S(t)$ is the vector of stock price processes) and portfolio processes $\pi(t), t \in [0, T]$ (which we assume to be square integrable progressively measurable with respect to the filtration $\mathbb{F}$), our framework of an investment problem under sustainability constraints might obviously require the modeling of

- the dynamics $D(t)$ of the cumulative demand of the customers for sustainable investments expressed in percent of their invested sum,
- the dynamics of sustainability ratings $R_i(t)$ of the different assets,
- and their possible influence on the dynamics of the asset prices.

We assume that the cumulative demand $D(t)$ will be expressed in the (volume-weighted) average rating demand, i.e. the rating $R(t)$ of the total wealth of the fund/the insurer at time t has to satisfy the *sustainability constraint*

$$R(t) \geq D(t) \quad \forall t \in [0, T] .$$

Further, we in particular assume that there is an accepted rating system for all assets involved in our investment universe.

Let, $X^\pi(t)$ denote the wealth process of a fund/an insurer that invests in various assets according to the portfolio strategy. Further, for simplicity, we assume that the utility function is given as $U(x) = \ln(x)$ which means that our investor considers the (expected) growth rate of wealth as the criterion to look at. We then arrive at the portfolio problem:

$$\max_{\pi(.)\in A(x)} E_{0,x}\left(U\left(X^\pi(T)\right)\right)$$
$$\text{such that } R(t) \geq D(t)\ \forall t \in [0, T]$$

Here, x is the initial wealth. $A(X)$ denotes the set of *admissible portfolio processes* that in addition to the integrability and measurability requirements stated above, have to satisfy

$$E_{0,x}\left(U\left(X^\pi(T)\right)^-\right) < \infty.$$

3 Some explicitly solved portfolio problems with sustainability constraints

We will present some examples of explicitly solvable portfolio problems below. For this, we start with the simple setting of a fund that offers its members the possibility to also invest in sustainable assets and will then look in more detail at problems relevant for a life insurer.

Let us also mention that we are not contributing to the vast literature of sustainable investment in the context of one-period portfolio optimization of Markowitz type or multi-criteria variants of it (see e.g. [11] or [2] and the references therein). Our main interest is to present examples of portfolio problems with explicit solutions that allow a clear interpretation and that hint at some specific features/possibilities of insurance companies to cope with various aspects of sustainable investments.

In particular, we are suggesting the use of continuous-time portfolio optimization as it will turn out that the actuarial reserve fund of a life insurer and its use in a continuous-time framework can play a central role in dealing with sustainability rating risks and product construction.

3.1 A first example: log-utility, constant sustainability rating, continuous-time demand

Let us start with a simple portfolio problem with a money market account and d stocks where we assume that the demand for sustainable investment and the sustainability ratings of the assets do not influence their price dynamics. To make this more precise, we explicitly require that

– each of the stocks has a constant sustainability rating $R_i \in [0, 1]$, the money market account is assumed to have a rating of $R_0 = 0$
– the demand process $D(t)$ for sustainable assets satisfies

$$D(t) \leq R^{\pi}(t) \quad \forall t \in [0, t] \tag{3}$$

and we also assume the *max-offer condition*

$$D(t) \leq R^* := \max\{R_1, \ldots, R_d\} \quad \forall t \in [0, t]\,. \tag{4}$$

– the stock prices are of the following form

$$dS_i(t) = S_i(t)\left(b_i dt + \sum_{j=1}^{d} \sigma_{ij} dW_j(t)\right).$$

To keep things simple, we use the log-utility function $U(x) = \ln(x)$ and consider the portfolio problem under sustainability constraints

$$\max_{\pi(.)\in A(x)} \mathbb{E}_{0,x}\left(\ln\left(X^{\pi}(T)\right)\right) \quad s.t.\ D(t) \leq R^{\pi}(t) \quad \forall t \in [0, T] \tag{5}$$

where we have implicitly assumed that the max-offer condition (4) will be cared for by the sales coordination, i.e. the demand will only be satisfied until $D(t) = R^*$ is reached.

Definition 1 The set of sustainable portfolios $A_S(x)$ is defined as the subset of the admissible portfolio processes $A(x)$ that in addition satisfy the sustainability constraints (3) and (4).

Application of Itô's formula to $\ln(X^\pi(T))$ and using the integrability requirements of an admissible portfolio process π lead to

$$\mathbb{E}_{0,x}\left(\ln\left(X^\pi(T)\right)\right) = \ln(x) + \mathbb{E}_{0,x}\int_0^T \left(r + \pi(t)'(b - r\underline{1}) - \frac{1}{2}\pi(t)'\sigma\sigma'\pi(t)\right) dt. \tag{6}$$

Pointwise maximization under the integral for fixed $t \in [0, T]$ and every $\omega \in \Omega$ yields the unconstrained optimal portfolio process π^* given by

$$\pi^*(t) = \pi^* := (\sigma\sigma')^{-1}\left(b - r\underline{1}\right) \tag{7}$$

with $\underline{1} = (1, \ldots, 1)' \in \mathbb{R}^d$. If the unconstrained optimal portfolio already satisfies the sustainability condition (3), then it is also the optimal sustainable portfolio. If this is not the case then elementary Lagrangian multiplier considerations—again applied to the integrand ω-wise for every $t \in [0, T]$—yield the following result:

Proposition 1 *Under the assumption* (4) *the optimal portfolio process for problem* (5) *is given by*

$$\pi_S^{opt}(t) = \begin{cases} \left(\sigma\sigma'\right)^{-1}\left(b - r\underline{1}\right), & \text{if } R(t)'\left(\sigma\sigma'\right)^{-1}\left(b - r\underline{1}\right) \geq D(t) \\ \left(\sigma\sigma'\right)^{-1}\left[\left(b - r\underline{1}\right) + \frac{D(t) - (b - r\underline{1})'(\sigma\sigma')^{-1}R(t)}{R(t)'(\sigma\sigma')^{-1}R(t)} R(t)\right], & \text{else.} \end{cases}$$

Remark 1 (Effects of the sustainability constraint for independent stocks) In the case of independent stocks, if the optimal sustainable portfolio satisfies the sustainability constraint exactly (i.e. we are in the second case of the proposition) then we can observe:

- Compared to the unconstrained optimal portfolio, the stock positions will not be decreased. However, they will only be increased for stocks with a strictly positive sustainability rating!
- Possible credits (i.e. a negative bond position) will only be increased in absolute value to buy additional sustainable stocks.

In the case of correlated stocks, the above remarks need not necessarily be correct. However, it can still happen that

- we will obtain an optimal negative bond position,
- a negative bond position can only be avoided by a reduction of positions in non-sustainable stocks.

In particular, for $d = 1$ we obtain

$$\pi_S^{opt}(t) = \begin{cases} D(t)/R(t), & \text{if } R(t)(b - r)/\sigma^2 < D(t) \\ (b - r)/\sigma^2, & \text{else.} \end{cases}$$

Example 1 (Independent assets, $d = 2$) While the example in the preceding remark is not extremely enlightening, we will present some interesting aspects in the setting of $d = 2$ independent, sustainable stocks, a fixed demand D and fixed ratings R_i. If π^* does not satisfy the sustainability constraint, then by the proposition we have to shift our portfolio components according to

$$\pi_1^* \to \frac{b_1 - r + cR_1}{\sigma_1^2}, \quad \pi_2^* \to \frac{b_2 - r + cR_2}{\sigma_2^2}, \quad c = \frac{D - \pi_1^* R_1 - \pi_2^* R_2}{R_1^2/\sigma_1^2 + R_2^2/\sigma_2^2}.$$

Note again that to satisfy the rating constraint, we need to increase our stock positions, i.e. increase the risky fraction of our investment. This goes along with a reduction in the bond—or in case of an insurer—a reduction of the investment in the actuarial reserve fund. The explicit representations of the optimal admissible portfolio components also show that stock positions with a non-positive sustainability rating stay constant and will in particular not be reduced.
To illustrate our findings with some numbers we choose the following figures:

$$D = 0.2, r = 0.01, b_1 = 0.03, b_2 = 0.04, \sigma_1 = 0.2, \sigma_2 = 0.3, R_1 = 0.1, R_2 = 0.3.$$

This then yields

$$\pi^* = (1/2, 1/3), \quad R = 0.1 * 1/2 + 0.3 * 1/3 = 0.15 < 0.2 = D, \quad c = 0.04,$$

i.e. the unconstrained optimal portfolio does not satisfy the sustainability constraints. As described above, we then obtain the optimal admissible sustainable portfolio as

$$\pi_1^* \to \frac{0.02 + 0.004}{0.04} = 0.6, \quad \pi_2^* \to \frac{0.03 + 0.012}{0.09} = 0.467.$$

Hence, it can easily be seen that both the portfolio return increases from 0.03 to 0.036 and the portfolio volatility increases from approximately 0.14 to 0.18 for the purpose of satisfying the sustainability constraint. If we in addition require a non-negative bond component, then the optimal admissible sustainable portfolio equals (0.5, 0.5).

Remark 2 (The ARF as fixed rate investment) If the bond component is given by the annual declaration of the actuarial reserve fund (ARF) of a life insurer, then it makes sense to assume that it also has a sustainability rating R_0. As a first consequence, we obtain a new representation for the portfolio sustainability rating as we have

$$R^{\pi}(t) = \left(1 - \underline{1}'\pi(t)\right) R_0 + \pi(t)' R = R_0 + \pi(t)' \left(R(t) - R_0 \underline{1}\right).$$

We can then solve the constrained portfolio problem again via a Lagrangian approach and obtain a modification of Proposition 1.

Proposition 2 *If under the assumption* (4) *the bond/actuarial reserve fund possesses a sustainability rating* $R_0 \geq 0$*, then the optimal portfolio process for problem* (6) *is given by*

$$\pi_S^{opt}(t) = \begin{cases} \left(\sigma\sigma'\right)^{-1}\left(b - r\underline{1}\right), \\ \quad \text{if } R_0 + (R(t) - R_0\underline{1})'\left(\sigma\sigma'\right)^{-1}\left(b - r\underline{1}\right) \geq D(t) \\ \left(\sigma\sigma'\right)^{-1}\left[\left(b - r\underline{1}\right) + \frac{D(t) - R_0 - (b - r\underline{1})'\left(\sigma\sigma'\right)^{-1}(R(t) - R_0\underline{1})}{(R(t) - R_0\underline{1})'\left(\sigma\sigma'\right)^{-1}(R(t) - R_0\underline{1})}(R(t) - R_0\underline{1})\right], \\ \quad \text{else} \end{cases}$$

Note that the interpretations are similar to those already given after Proposition 1, but now due to the subtraction of $R_0\underline{1}$, we can see that—at least in the case of independent stocks—positions of a stock with a sustainability rating below that of the actuarial reserve fund will be reduced if the unconstrained optimal portfolio is not admissible. We will highlight the effect of a positive rating of the actuarial reserve fund in the next example.

Example 2 (The case of a sustainable actuarial reserve fund) We consider the case of $D(t) = D$, two independent and non-sustainable assets (i.e. $R_1 = R_2 = 0$), but a sustainable ARF with $R_0 > 0$. Assuming that the unconstrained optimal portfolio is not admissible under sustainability constraints, we obtain the optimal sustainable portfolio as

$$\pi_1^* \to \frac{b_1 - r - cR_0}{\sigma_1^2}, \quad \pi_2^* \to \frac{b_2 - r - cR_0}{\sigma_2^2}, \quad c = \frac{D - R_0\left(1 - \pi_1^* - \pi_2^*\right)}{R_0^2} \frac{\sigma_1^2\sigma_2^2}{\sigma_1^2 + \sigma_2^2},$$

i.e. we look at the same portfolio problem if we replace the drift coefficients b_i by $b_i - cR_0$.

To illustrate our findings with some numbers we choose the following figures:

$$D = 0.2, r = 0.01, b_1 = 0.03, b_2 = 0.04, \sigma_1 = 0.2, \sigma_2 = 0.3, R_0 = 0.3, R_1 = R_2 = 0.$$

This then yields

$$\pi^* = (1/2, 1/3), \;\; R = 0.3 * 1/6 = 0.05 < 0.2 = D, \;\; c = 0.0462,$$

again, the unconstrained optimal portfolio does not satisfy the sustainability constraints. Now, we obtain the optimal admissible sustainable portfolio as

$$\pi_1^* \to 0.154, \;\; \pi_2^* \to 0.179, \;\; \pi_0^* = 2/3.$$

Hence, both the portfolio return decreases from 0.03 to 0.018 and the portfolio volatility decreases from 0.14 to 0.062 for the optimal sustainable portfolio. Further, as a consequence of the sustainability constraint, the optimal growth rate decreases from 0.02 to 0.0165.

3.2 A second example: how much sustainability risk can be compensated by a well-managed actuarial reserve fund?

At the beginning of this example we would like to highlight the difference between a sustainability constraint and an optimization criterion:

- A constraint—such as a sustainability constraint—can be seen as a promise or a contract that has to be fulfilled. Not satisfying the constraint is simply not an option.
- An optimization criterion is something that should be achieved as good as possible. Attaining a high value of the criterion can be seen as good work, attaining only a low one as mediocre or bad performance. In any case, it never is the equivalent of a contract that has to be fulfilled.

From this point of view towards constraints and optimality criteria, it becomes clear that we are in a *constraints first*-framework. This has also been seen in the foregoing examples where satisfying the constraint has been more important than getting the highest possible expected growth rate.

Keeping these remarks in mind, we now consider the situation when the actuarial reserve fund has a constant interest rate r on the time interval $[0, T]$ (i.e. we mainly look at a time horizon of one year until the next declaration of a constant rate happens) and a sustainability rating of R_0 that can (at least!) be kept at this level by suitably adapting its ingredients over time. In addition, the second investment opportunity is a fund that carries a risk of a rating change, i.e. at time T the rating of the fund can change according to

$$R \to R^\circ \quad \text{with} \quad R > R^\circ.$$

If such a kind of down rating will take place it might be possible that the sustainability constraint can no longer be satisfied by the insurance product made up of the positions in the ARF and this fund. As a consequence, a large portion of the fund has to be sold and it is highly likely that the insurer will suffer a big liquidity loss.

One can argue that this is not a new problem as we will also get a new constant declaration r° for the next time period. However, this is only relevant for the optimization criterion and not for the constraint. Thus, the optimal position can also be approximated slowly by shifting small amounts in the right direction without realizing a liquidity loss.

So, the central question is: How can a liquidity loss be avoided in the case of a down rating of the fund? There are some possible answers below of which some are more likely to happen than others. There is no liquidity loss

- if the position in the fund does not have to be changed, i.e. the optimal unconstrained portfolio is still sustainable,
- if there is an agreement with the fund provider that the fund will be taken back at the market price if it is down rated,
- if it is possible to compensate the down rating of the fund by increasing the sustainability rating of the actuarial reserve fund via a foreward-looking sustainability management.

As the first two cases can be seen as lucky incidents where there is nothing to do, the last case deserves a more precise description. The task is to manage the ARF in such a way that the sustainability condition for the whole portfolio is (nearly) always satisfied. There are at least two suggestions how to do that:

- A *worst-case approach* that consists of managing the ARF in such a way that at time T the portfolio rating is above $D(T)$ independent of the rating of the fund.
- A *monitoring approach* that consists of monitoring the actual sustainability status of the fund and then adjusting the ARF in an appropriate way. For this, one tries to estimate the conditional expectation of the sustainability rating of the fund at time T, $E(R(T)|\mathcal{F}_t)$ and managing the ARF such that with

$$\tilde{R}(t) := \min\{R, E(R(T)|\mathcal{F}_t)\}$$

we always have

$$R^{\pi}(t) = R_0(t) + \pi(t)(\tilde{R}(t) - R_0(t)) \geq D(t),$$

i.e. the sustainability rating of the ARF has to satisfy

$$R_0(t) \geq \frac{D(t) - \pi(t)\tilde{R}(t)}{1 - \pi(t)}$$

where we additionally assume that no short position in the ARF is allowed.

Both these approaches need additional research and specification as in the first one there are still a lot of degrees of freedom, while the second one needs a more precise description how the conditional expectation of the sustainability rating of the fund at the time T of the possible rating switch can be calculated.

Note also that the unconstrained optimal portfolio process will change at time T (if for simplicity we assume that rating change and the declaration time T of the fixed rate for the ARF coincide) due to the fact that we will get a new constant interest rate due to the new declaration.

Remark 3 (Constant r) Note that our actions of managing the ARF on the investment period $[0, T]$ under consideration do not affect the declared interest rate r as this has its reason in past performance. So, assuming that there is an opportunity for achieving a constant rate of return is still justified.

3.3 A third example: rebuilding the ARF

We consider the case that the prognosis for the demand $D(t)$ exceeds R^*, the maximal sustainability rating over all available funds. The natural way out of this situation is a rebuilding of the ARF with the aim to increase R_0, its sustainability rating. This can require spectacular actions such as buying wind parks, but also a less spectacular one such as a sustainable renovation of real estate or a slight change in the asset allocation.

However, we assume that increasing R_0 by such a rebuilding of the ARF also causes costs. These can be financed by a short term reduction of the declaration for the next year combined with a gained potential for an increase of the declaration in coming years. Let k be the costs and $r(k)$ the corresponding rate of declaration, $R_0(k)$ the corresponding sustainability rating, R_0^* the maximally achievable rating by a restructuring of the ARF with

$$\begin{aligned} R_0^* &> D(t) \forall\, t \in [0, T], \\ R_0(k) &= R_0 e^{-\gamma k} + R_0^* \left(1 - e^{-\gamma k}\right), \\ r(k) &= r e^{-\delta k}. \end{aligned}$$

Note in particular, that we have

$$R_0(0) = R_0, \quad R_0(\infty) = R_0^*, \quad r(0) = r > r(k), \quad r(\infty) = 0$$

and also

$$R(k) := R^{\pi}(k) := R_0(k)\left[1 - \pi_1 - \pi_2\right] + R_1\pi_1 + R_2\pi_2.$$

As we are in the log-utility setting, we know that for a fixed value of k we obtain the optimal sustainable portfolio as

$$\pi_S^{opt}(t) = \pi_S^{opt}(t; k) \\ = \begin{cases} \left(\sigma\sigma'\right)^{-1}\left(b - r(k)\underline{1}\right), \\ \quad \text{if } R_0(k) + \left(R(t) - R_0(k)\underline{1}\right)'\left(\sigma\sigma'\right)^{-1}\left(b - r(k)\underline{1}\right) \geq D(t) \\ \left(\sigma\sigma'\right)^{-1}\left[\left(b - r\underline{1}\right)\right. \\ \quad \left. + \frac{D(t) - R_0(k) - \left(b - r(k)\underline{1}\right)'\left(\sigma\sigma'\right)^{-1}\left(R(t) - R_0(k)\underline{1}\right)}{\left(R(t) - R_0(k)\underline{1}\right)'\left(\sigma\sigma'\right)^{-1}\left(R(t) - R_0(k)\underline{1}\right)}\left(R(t) - R_0(k)\underline{1}\right)\right], \text{ else.} \end{cases}$$

Remark 4 (Optimal choice of k?) What remains is to decide about the value k. For this, one can decide in the following steps:

- First, decide about the choice of the maximal fraction D^* of sustainable investment in the portfolio that should be offered to the customers in total. For this, an orientation could be the additional (potential) gain generated by more customers in relation to the needed amount of investment k.
- Determine k^* such that we have

$$R(k^*) \geq D(t)\ \forall t \in [0, T]$$

  and that k^* yields the optimal sustainable portfolio $\pi_S^{opt}(t; k^*)$ which leads to the highest expected growth rate among all k with $R(k) \geq D(t)$.

While the first constraint is a decision about the maximal offer of sustainable investment, the second one is a maximization requirement for the realizable expected growth rate for the sustainable rebuilding of the ARF.

Example 3 We will illustrate our findings again with some numbers and choose the following figures (with independent stocks):

$$D = 0.25, r = 0.01, b_1 = 0.03, b_2 = 0.04, \sigma_1 = 0.2, \sigma_2 = 0.3,$$
$$R_0 = 0.25,\ R_1 = 0,\ R_2 = 0.2.$$

This then yields

$$\pi^* = (1/2, 1/3),\ \ R = 0.25 * 1/6 + 0.2 * 1/3 = 0.1083 < 0.2 = D,\ \ c = 0.0891,$$

again, the unconstrained optimal portfolio does not satisfy the sustainability constraints. Now, we obtain the optimal admissible sustainable portfolio as

$$\pi_1^* \to -0.057,\ \ \pi_2^* \to 0.284,\ \ \pi_0^* = 0.773.$$

Hence, the optimal growth rate decreases from 0.02 to 0.0137. If we now want to optimize the rebuilding of the ARF with parameters

$$R_0^* = 0.45,\ \ \delta = 1, \gamma = 2$$

then we get an optimal choice of $k^* = 0.5206$ with $r(k^*) = 0.0059$ and $R_0(k^*) = 0.3795$, i.e. we have to reduce the declaration by approx. 40%, and get the optimal corresponding sustainable portfolio after (!) the rebuilding of the actuarial reserve fund as

$$\pi_1(k^*) = 0.202,\ \ \pi_2(k^*) = 0.294,\ \ \pi_0(k^*) = 0.504.$$

Note that

- The rebuilding of the ARF needs quite a *substantial reduction of the declaration* and yields *a higher sustainability rating of the ARF as is necessary*.
- The rebuilding allows for a *significantly higher investment in the stocks*, in particularly in the first one although it has a non-positive sustainability rating.
- The expected *growth rate after rebuilding* has significantly increased to 0.0161.

So, while at the first time being very costly, the rebuilding of the ARF has even paid out from an investment point of view. Increasing the maximum offer further to $D^* = 0.4$ then first yields a dramatic short position of $\pi_1^* = -0.646$ with an extremely poor expected growth rate of -0.007 (!). Again, a suitable rebuilding of the actuarial reserve fund results in a huge reduction of the declaration to just $r(k^*) = 0.0036$, an increase of the sustainability rating of the ARF to $R_0(k^*) = 0.4235$, to optimal portfolio values of

$$\pi_1(k^*) = -0.068,\ \ \pi_2(k^*) = 0.234,\ \ \pi_0(k^*) = 0.834,$$

and an expected growth rate of 0.008 which is much higher than before the rebuilding. Note, however, that now the sustainability requirement which is close to the absolutely achievable one by rebuilding does not allow investment in the non-sustainable asset.

We hope that the example has demonstrated possibilities and impossibilities of the rebuilding process of the ARF.

Remark 5 (Extension to other utility functions) We have so far concentrated on the logarithmic utility functions to obtain explicit results that are fully proved. However, we believe that our results can be extended to the power utility function and the exponential utility function, too. Of course, we might need a more sophisticated way to show optimality (such as using a constrained HJB-equation).

Remark 6 (Losses by sustainability constraints) So far, adding sustainability requirements only led to an overall performance of the optimal portfolios (with or without the rebuilding process of the ARF). This, however, has been clear before solving the portfolio problems, as a constrained maximization problem cannot yield a higher optimal value of the criterion than an unconstrained one. The main problem with the setting so far is that we have not yet modeled the impact of the climate crisis, the desire for a healthier environment and the tendency to get rid of shares of companies that do not seem to have a bright future due to various reasons (including their high polluting and energy intensive production process) or also due to expectable future increasing charges of pollution. Taking these side aspects into consideration might change the effect of the sustainability requirement in the portfolio problem. We will take this up by introducing new possible price models in the next section.

4 Modeling and risk management challenges for investment with sustainability constraints

In the foregoing sections we mainly assumed independence between the asset price dynamics, the demand process for sustainable assets and the sustainability rating of the assets. Further, we often benefited from constant drift terms of the stock prices. In this section, we will have a look at some modeling challenges for the ingredients of our portfolio problem and will also state some conceptual (risk) management challenges which are caused by the presence of sustainable assets and sustainability constraints.

4.1 More sophisticated modeling

We start by proposing some more sophisticated models (compared to those of the foregoing section) for the evolution of

- the demand processes for sustainable investments,
- the sustainability rating process for assets,
- the asset price modeling with possible dependence of the sustainability rating and changing drift parameters.

Our main motivations behind the proposed models will be given when we consider them in detail. We will further present some future research challenges that shall motivate future research.

The demand process for sustainable assets. We believe that due to the challenges of the climate change the demand for sustainable assets will gain a stable value in the future. Of course, this might depend on the performance of those assets, too. But also with respect to this issue, we believe that successful sustainable investments will gain a stable performance, while for some there may be the risk to completely disappear from the market.

Having said this, a first suggestion for the evolution of the demand process for sustainable assets has the following deterministic form

$$D(t) = \hat{D}\left(1 - e^{-\delta t}\right) + D_0 e^{-\delta t}, \tag{13}$$

i.e. we assume a tendency to move from the initial demand D_0 to the stable demand $\hat{D}$ where it might be hard to specify the stable demand for sure, but given the specification of the dynamics in Equation (13), we know already after a very short time if we have $D_0 < \hat{D}$ or $D_0 \geq \hat{D}$. Given this modeling, the portfolio optimization examples of the foregoing chapter can be solved with obvious minor modifications.

If we want to introduce a stochastic component in this setting a mean-reverting process is the natural generalization of the above deterministic setting. As we must have $0 \leq D(t) \leq 1$, an obvious candidate is the Jacobi process given by

$$dD(t) = \delta\left(\hat{D} - D(t)\right)dt + \sigma\sqrt{D(t)(1 - D(t))}dW_D(t), \quad D(0) = D_0$$

(with $0 \leq \hat{D} \leq 1$) where $W_D(t)$ is a Brownian motion that can be correlated to the ones driving the different asset price evolutions (see e.g. [3] or [1] for details on the Jacobi process and applications to interest rate and electricity demand modeling). While such a stochastic model looks appealing, it might be doubtful if the technical complications corresponding to it will be compensated by the modeling benefits.

Future research challenge 1: We consider the *use of the Jacobi process* as ingredient in the optimal investment problem with sustainable assets as one of the challenges for future research that we would like to state here, especially if we have an unknown mean-reversion level of $\hat{D}$.

The rating process for sustainable assets. One can imagine about a great variety of ratings that can range from a 0-1-type rating (*sustainable/not sustainable*) to the Bloomberg/Heinke rating with a continuous range in [0, 1] that we have proposed as the basis of our work. However, all those ratings should contain the possibility of improvement or decline of the rating over time. Thus, it seems mandatory to model the sustainability rating process $R_i(t)$ of each asset i.

A rating process that fits best for the use of sustainable assets in a life insurer's portfolio is constant for the same period as the declared interest rate on the ARF. If for simplicity we assume these periods to start at January 1 and end at December 31 then this rating

$$R_i(t) = R^{(j)}, \quad t \in [k, k+1), \quad R^{(j)} \in [0, 1],$$

would on one hand lead to the possible problems of shifting big amounts of money from one asset to another at January 1 of year $k + 1$. On the other hand it shows the big advantage of the existence of the ARF of a life insurer: given that the life insurer is able to monitor or estimate the sustainability status of the asset between k and $k + 1$, appropriate reallocation in the ARF can be taken to avoid this risk of a rating change. More precisely, the life insurer should not be surprised by the change of the rating and therefore might take small actions over a longer time horizon rather than having to suffer from liquidity discounts for selling a big position and liquidity premia for buying a large one on January 1.

Of course, a continuous rating where $R_i(t)$ can again be modeled as a Jacobi process would be a kind of solution, but it might not be possible to assign and monitor this in practical applications.

Future research challenge 2: Independent on the modeling of the evolution and the type of the sustainability rating, we can state another major challenge of future research, the *prediction of the future sustainability rating* and the *appropriate hedging actions* against it.

Advanced models for the evolution of the price of sustainable assets. While the effect of modeling the demand process and/or the rating process for sustainable assets as stochastic processes seems to be more or less straight forward, their impact on the modeling of the price evolution of corresponding sustainable assets is not clear at all and leaves many opportunities.

We will therefore motivate some suggestions and hint at the possible challenges from both a statistical and a portfolio optimization point of view:

Model 1: *Rating-dependence of the market coefficients.* A first simple asset price model with drifts admitting dependence from the sustainability rating is

$$dS_i(t) = S_i(t)\left[(r + \lambda_i(R_i(t)) + \gamma_i(R_i(t)))dt + \sum_{j=1}^{i} \sigma_{ij} dW_j(t)\right].$$

Here, both the *risk premium* $\lambda_i(R_i(t))$ and the *costs* $\gamma_i(R_i(t)) \in \mathbb{R}$ of keeping the asset at the aimed sustainability rating (such as e.g. a better and environmentally friendly maintaining of the production line of the company, payment of fair salaries, better working conditions, etc.) depend on the rating of asset i. Of course, this dependence already gives a wide modeling range. A simple version of a non-constant drift consists of piece wise constant values of λ_i, γ_i as a consequence of piece wise constant ratings that are reviewed regularly once a year.

From a statistical point of view the cost intensity γ_i could be predicted quite accurately by the company itself, while the estimation of the risk premium corresponds to the standard estimation problem of the stock price drift. Thus, in this piece wise constant parameter setting, the optimization problem can be solved as in the examples of the foregoing chapter, but with minor modifications.

What however causes a much bigger challenge is the possibility of the change of the sustainability rating at a random time.

Model 2: *A two-valued stock price drift.* Here, we look at a simple model for a randomly changing drift which is motivated by the class of reduced form models in credit rating (see e.g. [4]). For this, we assume that the drift of the sustainable asset has the form of $r + \lambda_i(t)$ with

$$\lambda_i(t) = \begin{cases} \lambda_i^{(1)} & \text{for } t < \tau_i, \\ \lambda_i^{(2)} & \text{for } t \geq \tau_i, \end{cases}$$

and where $\tau_i \sim Exp(w_i)$ is an exponentially distributed stopping time. If one looks at the relation (6), one is tempted to say that $E(\lambda_i(t))$ takes over the role that the constant λ_i in $b_i = r + \lambda_i$ would play in the optimal solution. However, this is not the case as the ω-wise maximum inside the brackets is still attained for $\pi^*(t)$ as in representation (7), i.e. in the case of just one stock we still have

$$\pi^*(t) = (b(t) - r)/\sigma^2 = \lambda_i(t)/\sigma^2.$$

As long as the risk premium stays in the lower values, the mean of $\lambda_i(t)$ is of course relevant for computing the expected growth rate, but not for the optimal strategy.
Here, it is also the question if the non-predictable drift change is related to a change in the sustainability rating of the stock or not. In the latter case, there is no need for immediately making a big reallocation of funds. The non-optimal portfolio position is likely to be not so bad as the losses caused by limited liquidity of the market. If, however, the change in the drift is due to a change of the sustainability rating then this can have consequences:

- if the change is a downgrade, then additional sustainable investment has to be made, at least if the sustainability condition has been active before the change. Depending on the relation between the market coefficients of the ARF and the stock this can either be additional purchases or sales of the stock.
- if the change is an upgrade then additional purchases of the stock are possible to increase the portfolio growth rate, but they do not necessarily have to be made in the form of a big reallocation. They can be performed over time as normal, infinitesimal transactions.

Example 4 Simple examples that illustrate the foregoing consequences are given in the setting of just the ARF and one sustainable stock. For this, let us assume that we fix the following parameters

$$D = 0.18, r = 0.02, \sigma = 0.2.$$

and additionally consider the two different sets of parameters below:

$$R_0 = 0, R_1^{(1)} = 0.2, R_1^{(2)} = 0.15, \lambda_1^{(1)} = 0.04, \lambda_1^{(2)} = 0.03,$$
$$R_0 = 0.2, R_1^{(1)} = 0.15, R_1^{(2)} = 0.2, \lambda_1^{(1)} = 0.03 = \lambda_1^{(2)}.$$

For the first set, before the change in the drift the sustainability constraint is already satisfied by the optimal unconstrained fraction invested in the stock of $\pi^*(t) = 1$. In this setting, the rating change comes with a decreasing drift and leads to an optimal unconstrained portfolio process after the change of $\pi^*(t) = 0.75$. However, the value to satisfy the sustainability constraint now equals $\pi(t) = 1.2$. Thus, the necessary change of the portfolio requires a big reallocation of funds, a negative position in the

ARF and also comes with a decrease of the growth rate. To avoid the liquidity loss of the lump-sum reallocation, the insurer could have already chosen to hold the non-optimal position of $\pi(t) = 1.2$ before the change of the rating or could have limited the maximum offer of sustainable assets. For the second parameter set, the unconstrained optimal portfolio before the change to $\pi^*(t) = 0.75$ does not satisfy the sustainability constraint. Instead, the constrained portfolio with the highest growth rate before the rating change equals $\pi(t) = 0.4$. However, after the rating change, $\pi^*(t) = 0.75$ becomes admissible with respect to the sustainability constraint. The insurer is now free how to move in the direction of this portfolio and can weigh the gain in growth rate against the loss by paying liquidity premia for a big reallocation.

Model 3: *A demand-dependent drift function.*
Of course, one should also consider the fact that an extraordinary demand for a certain sustainable asset may cause an extraordinary increase of the price. This can be modeled by a temporary extra drift parameter that will approach zero if the demand for the asset has reached a stable level. A possible price modeling approach can be

$$dS(t) = S(t)\left[(r + \lambda + \lambda_1(\hat{D} - D(t)))dt + \sigma dW(t)\right],$$
$$dD(t) = \delta\left(\hat{D} - D(t)\right)dt + \sigma\sqrt{D(t)(1 - D(t))}dW_D(t)$$

with the two Brownian motions $W(t)$ and $W_D(t)$ possibly being correlated.

Future research challenge 3: Finding a modeling approach for stock price dynamics with additional drift in a phase of extraordinary temporarily demand clearly is a challenge, in particular, if we look at a model that in addition allows the estimation/calibration of all coefficients. Of course, we do not consider our suggestion above as the only suitable one.

4.2 Some conceptual challenges

In the second part of this section, we now look at conceptual and risk management challenges of which we believe that it is relevant to solve them in future research.

Conceptual challenge 1: *Separate ARFs or only one ARF?*
The question of having two separate actuarial reserve funds is controversially discussed in the actuarial community. The proposed models vary between having just one ARF for all customers independent of their attitude towards sustainable investment as one extreme and having two completely separate ARFs for traditional customers and customers who explicitly want to invest parts of their contributions into sustainable assets as the other extreme. However, we have seen in Sect. 3.3 that creating a (partly) sustainable ARF is an innovative and particularly useful tool. Thus, the ARF—as a trademark of a life insurer—can get an additional role, a fact which even strengthens its importance.
Of course, it seems to be attractive to completely separate the sustainable and the non-sustainable part of the ARF and then mix the fractions of the two ARFs according to the taste of each group of customers. To do this, one should first find the optimal

portfolio process *without* taken into account the sustainability constraint. Thus, the inclusion of the sustainable assets that are needed to satisfy the sustainability constraint are not even used for diversifying the risk of the investors that do not explicitly want to include them. As we will see in our example below, such a case can exist.

In this case, it is easy to build up two separate ARFs, one consisting only of the assets that enter the unconstrained portfolio process. The second ARF then contains all the assets that enter the optimal constrained portfolio.

Example 5 An example to illustrate this can be given for two correlated (!) stocks with equal drift term, a non-sustainable money market account and a demand of sustainable assets of $D = 0.2$. More precisely, we have the following parameters:

$$r = 0.02,\ R_0 = R_1 = 0,\ R_2 = 0.3,\ b_1 = b_2 = 0.05,$$
$$\sigma_{11} = 0.2,\ \sigma_{12} = 0,\ \sigma_{21} = 0.2,\ \sigma_{22} = 0.2,$$

By explicit computations, we obtain that the unconstrained optimal portfolio process $\pi^*_{unconst}$ equals

$$\pi^*_{unconst} = (0.75, 0).$$

The optimal portfolio process under the sustainability constraint π^*_{sus} is given by

$$\pi^*_{sus} = (0.083, 0.667).$$

Note that in both portfolio processes we invest three quarters of our wealth in stocks. However, the second one leads to a much higher volatility. This is due to the fact that the second stock has a higher volatility than the first one, is also positively correlated with the first one, and the first one is only used in a small amount.

Having these two ARFs highlights the problem that we face in having two types of customers. The ones that are not interested in sustainability aspects at all would definitely go for their full investment in the first ARF as it yields a higher expected growth rate of $Growth_1 = 0.03125$ compared to $Growth_2 = 0.02236$. This is due to the higher volatility of the optimal portfolio process under sustainability constraints. Of course, the second ARF starts from scratch and is thus too small for representing a big collective. It can be seen as a special fund that has a sustainability rating of $R_{ARF_2} = 0.2$. The alternative to have only one ARF and to offer it as an ARF with the same sustainability rating might put old customers off due to the higher volatility. On the other hand it allows to offer a product with a sustainability rating of $R_{ARF_{total}} = 0.2$. More research on this aspect shall be performed in the future.

This should then also deal with the fact that usually sustainable assets already enter the unconstrained optimal portfolio simply for reasons of diversification.

Conceptual challenge 2: *Insurance solutions against a sustainability rating decline?* One of the main problems of the occurrence of a bad sustainability rating change (i.e. a change that requires the (immediate) liquidation of a big position in some asset in the actual portfolio) is a drop of the portfolio value by having to accept liquidity discounts on the sale side and liquidity premiums on the purchase side. We can also reformulate this challenge as how to avoid these costs.

The question is if insurance companies will open up a new branch of business to offer insurance against sudden rating changes. A possible way to hedge such an insurance can again be the use of the ARF. This time the insurer can think about monitoring the sustainability status of this product under rating risk, and take suitable actions to already sell non-sustainable products to be ready to replace them by the product under rating risk. Of course, the exact strategy how to do this and how to price such a product requires theoretical research and also a sustainability concept for hedging.

Conceptual challenge 3: *A portfolio problem with a continuous inflow process.*

So far, our portfolio problem was mainly a static one. We have been setting up a wealth process that satisfies the sustainability constraint and thus have created a product that performs optimal while satisfying the sustainability constraint.

However, a continuous demand process for sustainable assets should normally also go in line with a continuous capital inflow to the fund/the insurance company. Modelling this issue and its impact on the final wealth is an interesting subject.

Conceptual challenge 4: *Utility from final wealth versus utility from dividends.*
Until now, dividends have not been introduced into our model. One can give a theoretical justification in saying that the use of continuous price processes implicitly implies that the dividend is directly re-invested in the corresponding stock again and thus compensates the price drop at the dividend payment time.

Thus, if one wants to realize gains at a fixed time point—say the end of the year—then one has to either sell corresponding parts of the stocks that correspond to the ones that have been bought with the dividend money or one has to assign them to a separate account. With this the sustainable fund gets a mechanism that assigns it the character of an ARF. In this way, dividends can take over the role of coupons in a classical ARF.

On the other hand, structural type of investments such as solar or wind parks or buildings might be interpreted as a mixture of classical stocks and bond portfolios. While there is still some randomness in their actual annual performance, they are able to generate—at least locally—much more stable and predictable gains than shares of a company. On the other hand, they cannot be traded as liquidly as shares.

The modelling of a price or a dividend type process of structural investments and the construction of an ARF-type to generate a stable income for the customers are challenges worth to look at.

Conceptual challenge 5: *Global or individual bounds on sustainability demand.*
If an insurer has an ARF with a sustainability rating of say $R_0 = 0.2$ then a classic with participating life insurance can be regarded as a special fund that invests 20% of the contributions (ignoring costs for the moment) into sustainable assets. This would correspond to a global view. If, however, the insurer decides to use the ARF to offer classic life insurance products with different rates of sustainability, then in principle every rate between 0 and 1 can be offered as long as there are still enough sustainable parts in the ARF.

Such a strategy is much harder to manage as it needs exact book keeping of each individual customer and also requires a an individual strategy for assigning the investment gains to the customers. Thus, although they seem to be attractive at first sight, using and managing individual ("local") sustainability bounds, definitely is a challenging task.

5 Conclusion

With their ability to invest big amounts even in illiquid positions and hold them for a long time, life insurers and pension funds seem to be the perfect partners in the transformation of today's industrial societies towards a more ecologically oriented form. For this, however, to be a success the investment process needs some additional ingredients.

The first ingredients to gain the trust of the customers is a widely accepted and easy to understand ESG- or sustainability rating for the investment possibilities. With such a rating the customers' demand for sustainable investment can then enter the optimal portfolio problem of the pension fund or the life insurer as a constraint.

We have demonstrated some consequence of the presence of such a constraint in the portfolio optimization. We have also highlighted the particular role of having a (locally) fixed rate investment with a non-zero sustainability rating. This role can be attained by the actuarial reserve fund of an insurance company. Together with the one-year-delay with which the investment results enter the declaration, an insurance company can even deal with the risk that an investment will get a downgrade with respect to its sustainability rating.

However, to completely cope with the new situation of the sustainability rating entering investment decisions, new challenges for future research are arising. We have therefore stated some of them and hope to have motivated more research in this direction.

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Conflict of interest We declare that we have no conflict of interest.

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