

## On Different Formulations of Chiral Bosons

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### Abstract

It is shown, that recently constructed PST Lagrangians for chiral supergravities follow directly from earlier Kavalov–Mkrtchyan Lagrangians by an Ansatz for the  $\theta$  tensor by expressing this in terms of the PST scalar. The susy algebra which included earlier  $\alpha$ –symmetry in the commutator of supersymmetry transformations, is now shown to include both PST symmetries, which arise from the single  $\alpha$ –symmetry term. The Lagrangian for the 5-brane is not described by this correspondence, and probably can be obtained from more general Lagrangians, possessing  $\alpha$ –symmetry.

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# 1 Introduction

Among fields appearing in modern higher-dimensional theories are the (anti)–self–dual tensor fields, describing the representations of corresponding little groups with the constraint of (anti)–self–duality. Such tensors have a rank, corresponding to dimensionality and signature of space–time, and satisfy first–order field equations – the condition of self–duality of their field strength. In space–time with one time dimension the duality condition is possible in dimensions 2, 6, 10, etc. In 2 dimensions the fields are the chiral scalars, used e.g. in heterotic string theories [1]. In dimensions 6 and 10 the first theories considered, including as a necessary part a self–dual (or anti–self–dual) tensor, were supergravities [2], particularly 10 dimensional  $\mathcal{N} = 2b$  supergravity – one of two maximal supergravities (another one is 11d  $\mathcal{N} = 1$  supergravity [3]), dimensional reduction of which to 4d gives maximally extended  $\mathcal{N} = 8$  supergravity [4]. Recently another theory with self–dual field attracted much attention – the six–dimensional theory, known as 5-brane theory [5][6], which is an important feature of M–theory. The unique property of these fields is their contribution to the gravitational anomaly [7], the only contribution coming from bosonic fields. This property is intimately connected with the fact, that it is impossible to write down the Lagrangian for such a field in the usual way – and correspondingly the regularization of quantum theory meets a difficulty, which eventually leads to the appearance of the anomaly.

The problem of construction of Lagrangians for dual tensors, and eventually Lagrangians for theories mentioned above was addressed in a number of papers. Several methods were suggested for the construction of Lagrangians, among which are that of Siegel [8], the infinite auxiliary field method [9], and the recently invented PST formalism [10]. The generalization of the first one was used in the papers of Kavalov and Mkrtychyan about ten years ago[11], in which Lagrangians for all chiral supergravities were constructed for the first time. Recently the equivalence of the PST and infinite auxiliary field methods was claimed [12], and the Lagrangians for supergravities in the PST formalism were constructed [13]. The aim of the present paper is to show, that Lagrangians of ref. [10] follow directly from those of ref. [11] from an ansatz for the Lagrangian multiplier  $\theta_{\mu\nu\lambda,\sigma\rho\delta}^{++}$ , and that the symmetry algebra, which includes the so–called  $\alpha$ –symmetry, transforms into the algebra, containing specific PST symmetries, the origin of these terms being

exactly the  $\alpha$ -symmetry terms. The correspondence with the method of ref. [9] can also be established. In the following the detailed proof is given of the equivalence of the Lagrangians and the relations between the symmetry transformations. Section 2 is devoted to definitions and the description of properties of self-dual tensors. Sections 3 and 4 present the Lagrangians of Kavalov and Mkrtychyan, as well as  $\alpha$ -symmetry and the PST formulation. The ansatz for the transformation of one theory into another is presented in Sect. 5, together with the transformation of symmetries. The algebra of symmetries is considered in the next Section, where the appearance of two PST symmetries from a single  $\alpha$ -symmetry transformation is demonstrated. In Section 6 we summarise our results and make concluding remarks.

## 2 Definitions

We define the antisymmetric second rank tensor  $A_{\mu\nu}$  in  $d = 5 + 1$  dimensions with self-dual field strength  $F_{\mu\nu\lambda}$  by the following relations:

$$A_{\mu\nu} = -A_{\nu\mu}, \quad \mu, \nu = 0, 1, 2, 3, 4, 5. \quad (1)$$

$$F_{\mu\nu\lambda} = \partial_\mu A_{\nu\lambda} + \partial_\nu A_{\lambda\mu} + \partial_\lambda A_{\mu\nu}, \quad (2)$$

$$F_{\mu\nu\lambda}^\pm = \frac{1}{2} \left( F_{\mu\nu\lambda} \pm \frac{1}{6} \varepsilon_{\mu\nu\lambda\sigma\rho\delta} F^{\sigma\rho\delta} \right), \quad (3)$$

$$F_{\mu\nu\lambda}^- = 0, \quad (4)$$

We use (anti)symmetrization with unit weight, for example:

$$S_{(\mu\nu)} = \frac{1}{2} (S_{\mu\nu} + S_{\nu\mu}), \quad (5)$$

$$T_{[\mu\nu]} = \frac{1}{2} (T_{\mu\nu} - T_{\nu\mu}). \quad (6)$$

We can then prove the following identities valid for arbitrary antisymmetric third rank tensors, a symmetric traceless tensor  $\theta_{\mu\nu}$  and an antisymmetric tensor  $\Lambda_{\mu\nu}$ :

$$K_{\mu\nu\rho}^\pm H^{\pm\mu\nu\rho} = 0, \quad K_{\mu\nu\rho}^- H^{+\lambda\nu\rho} + K_{\nu\rho\mu}^{-\lambda} H_\mu^{+\nu\rho} = \frac{1}{3} \delta_\mu^\lambda K_{\alpha\nu\rho}^- H^{+\alpha\nu\rho}, \quad (7)$$

$$K_{\alpha[\nu\rho}\theta_{\lambda]}^{\pm} = \left(K_{\alpha[\nu\rho}\theta_{\lambda]}^{\pm}\right)^{\mp}, \quad K_{\mu\nu\rho}^{\pm}H^{\pm\lambda\nu\rho} - K_{\nu\rho}^{\pm\lambda}H_{\mu}^{\pm\nu\rho} = 0, \quad (8)$$

$$K_{\alpha[\nu\rho}\Lambda_{\lambda]}^{\pm} = \left(K_{\alpha[\nu\rho}\Lambda_{\lambda]}^{\pm}\right)^{\pm}, \quad K_{\mu\nu\rho}^{\pm}H^{\pm\mu\lambda\sigma} + K_{\mu}^{\pm\lambda\sigma}H_{\nu\rho}^{\pm\mu} = 2\delta_{[\nu}^{[\lambda}K_{\rho]\alpha\beta}^{\pm}H^{\pm\sigma]\alpha\beta}, \quad (9)$$

$$K_{\mu\nu\rho}^{-}H^{+\mu\lambda\sigma} - K_{\mu}^{-\lambda\sigma}H_{\nu\rho}^{+\mu} = \frac{1}{3}K_{\alpha\beta\gamma}^{-}H^{+\alpha\beta\gamma}\delta_{\nu}^{[\lambda}\delta_{\rho}^{\sigma]} - 2\delta_{[\nu}^{[\lambda}K^{-\sigma]\alpha\beta}H_{\rho]\alpha\beta}^{+}, \quad (10)$$

$$K^{-\mu\nu\rho}H_{\lambda\sigma\delta}^{+} + K_{\lambda\sigma\delta}^{-}H^{+\mu\nu\rho} = \delta_{\lambda}^{[\mu}\delta_{\sigma}^{\nu]}\delta_{\delta}^{\rho]}K_{\alpha\beta\gamma}^{-}H^{+\alpha\beta\gamma} - 9\delta_{[\lambda}^{[\mu}\delta_{\sigma}^{\nu]}K_{\delta]\alpha\beta}^{-}H^{+\rho]\alpha\beta} + 9\delta_{[\lambda}^{[\mu}K_{\sigma\delta]\alpha}^{-}H^{+\nu\rho]\alpha}, \quad (11)$$

$$K^{\pm\mu\nu\rho}H_{\lambda\sigma\delta}^{\pm} - K_{\lambda\sigma\delta}^{\pm}H^{\pm\mu\nu\rho} = 9\delta_{[\lambda}^{[\mu}\delta_{\sigma}^{\nu]}K_{\delta]\alpha\beta}^{\pm}H^{\pm\rho]\alpha\beta} - 9\delta_{[\lambda}^{[\mu}K_{\sigma\delta]\alpha}^{\pm}H^{\pm\nu\rho]\alpha} \quad (12)$$

### 3 The $\alpha$ -symmetry formalism for chiral bosons

We consider the following action, introduced in the work of Kavalov and Mkrtchyan [11], which involves tensor  $A_{\mu\nu}$  and the sixth-rank tensor  $\theta_{\mu\nu\lambda,\sigma\rho\delta}^{++}$ , the latter being self-dual over each set of three indices:

$$S_{KM} = \int d^6x \left\{ -\frac{1}{6}F_{\mu\nu\lambda}F^{\mu\nu\lambda} + \frac{1}{3}\theta_{\mu\nu\lambda,\sigma\rho\delta}^{++}F^{-\mu\nu\lambda}F^{-\sigma\rho\delta} \right\} \quad (13)$$

Here

$$\theta_{\mu\nu\lambda,\sigma\rho\delta} = \theta_{[\mu\nu\lambda],[\sigma\rho\delta]} = \theta_{\sigma\rho\delta,\mu\nu\lambda}, \quad (14)$$

$$\theta_{\mu\nu\lambda,\sigma\rho\delta}^{++} = P_{\mu\nu\lambda}^{+ \mu_1\nu_1\lambda_1} P_{\sigma\rho\delta}^{+ \sigma_1\rho_1\delta_1} \theta_{\mu_1\nu_1\lambda_1,\sigma_1\rho_1\delta_1}, \quad (15)$$

$$P_{\mu\nu\lambda}^{\pm \mu_1\nu_1\lambda_1} = \frac{1}{2} \left( \delta_{\mu}^{\mu_1} \delta_{\nu}^{\nu_1} \delta_{\lambda}^{\lambda_1} \pm \frac{1}{6} \varepsilon_{\mu\nu\lambda}^{\mu_1\nu_1\lambda_1} \right) \quad (16)$$

The tensor  $\theta$  serves as a Lagrange multiplier. From the Lagrangian of (13) we obtain the following equations of motion:

$$F_{\mu\nu\lambda}^{-}F_{\sigma\rho\delta}^{-} = 0, \quad (17)$$

$$\partial_{\mu} \left( F^{-\mu\nu\lambda} - \theta^{++\mu\nu\lambda,\rho\sigma\delta} F_{\rho\sigma\delta}^{-} \right) = 0 \quad (18)$$

These equations are equivalent to the self-duality-condition:

$$F_{\mu\nu\lambda}^{-} = 0 \quad (19)$$

and do not impose any restriction on the auxiliary field  $\theta_{\mu\nu\lambda,\sigma\rho\delta}^{++}$ . In addition to the usual gauge invariance:

$$\delta A_{\mu\nu} = 2\partial_{[\mu}\Lambda_{\nu]} \quad (20)$$

$S_{KM}$  is also invariant with respect to the so-called  $\alpha$ -symmetry with vector parameter  $\alpha^\rho$  defined by:

$$\delta(\alpha)A_{\mu\nu} = \alpha^\rho \left( F_{\rho\mu\nu}^- + \theta_{\rho\mu\nu,\lambda\sigma\delta}^{++} F^{-\lambda\sigma\delta} \right) \quad (21)$$

$$\delta(\alpha)\theta_{\mu\nu\lambda}^{++\sigma\rho\delta} = \left( P_\theta^+ A P_\theta^- \right)_{\mu\nu\lambda}^{\sigma\rho\delta} + \left( P_\theta^+ A P_\theta^- \right)_{\mu\nu\lambda}^{\sigma\rho\delta} - \alpha^\gamma \partial_\gamma \theta_{\mu\nu\lambda}^{++\sigma\rho\delta} \quad (22)$$

$$A = -\frac{3}{2}\delta_\mu^\sigma \delta_\nu^\rho \partial_\lambda \alpha^\delta, \quad \left( P_\theta^\pm \right)_{\mu\nu\lambda}^{\sigma\rho\delta} = P_{\mu\nu\lambda}^{\pm\sigma\rho\delta} \mp \theta_{\mu\nu\lambda}^{++\sigma\rho\delta}. \quad (23)$$

Using this approach for the Lagrangian of self-dual fields, in refs. [11] the supersymmetric Lagrangians were constructed for all supergravities, containing such fields in their supermultiplets (and even non-chiral supergravities were presented in such a form). The  $\alpha$ -symmetry (21) is maintained in these Lagrangians, and plays an important role in the closure of the algebra of symmetries of the theory. In particular the  $\alpha$ -part appears on the r.h.s of the commutator of two local supersymmetry transformations in all supergravities [11]:

$$\begin{aligned} [\delta(\varepsilon_2), \delta(\varepsilon_1)] &= \delta(diff) + \delta(Lorentz) + \delta(gauge) + \delta(\alpha) \\ &+ \delta(susy) + (eq. of motion) \end{aligned} \quad (24)$$

The explicit expressions depend, of course, on the specific theory considered, but the algebra (24) remains the same. Moreover, the parameter of  $\alpha$ -symmetry is always equal to the parameter of the general coordinate transformation:  $\alpha^\mu = \xi^\mu$ . As an example and for further use, we quote here some expressions for the simplest case of  $d = 6$ ,  $\mathcal{N} = 2$  chiral supergravity (notation as in [11]):

$$\alpha^\mu = \xi^\mu = \bar{\varepsilon}_1^a \gamma^\mu \varepsilon_{2a}$$

The susy transformation of  $\theta$  is:

$$\delta(\varepsilon)\theta_{\alpha\beta\gamma}^{++\alpha'\beta'\gamma'} = \left( P_\theta^+ E P_\theta^- \right)_{\alpha\beta\gamma}^{\alpha'\beta'\gamma'} + \left( P_\theta^+ E P_\theta^- \right)_{\alpha\beta\gamma}^{\alpha'\beta'\gamma'} \quad (25)$$

$$E = \frac{3}{2}\delta_\alpha^{\alpha'} \delta_\beta^{\beta'} e_\gamma^\mu \delta(\varepsilon) e_\mu^{\gamma'} \quad (26)$$

## 4 The Pasti-Sorokin-Tonin formulation of the chiral boson

The action constructed in ref. [10] is:

$$S_{PST} = \int d^6x \left\{ -\frac{1}{6} F_{\mu\nu\lambda} F^{\mu\nu\lambda} + \frac{2}{(\partial_\rho a \partial^\rho a)} \partial^\mu a F_{\mu\nu\lambda}^- F^{-\sigma\nu\lambda} \partial_\sigma a \right\} \quad (27)$$

where the scalar field  $a$  is an auxiliary field, analogous to  $\theta$  of the previous Section. This action is invariant under the following local gauge transformation:

$$\delta_1(\varphi) A_{\mu\nu} = \frac{2\varphi(x)}{(\partial_\rho a \partial^\rho a)} F_{\mu\nu\lambda}^- \partial^\lambda a, \quad \delta_1 a = \varphi(x) \quad (28)$$

$$\delta_2(\phi_\nu) A_{\mu\nu} = \partial_\mu a \phi_\nu - \partial_\nu a \phi_\mu, \quad \delta_2 a = 0 \quad (29)$$

As in the previous Section, there is only one independent equation of motion, in this case that of  $A_{\mu\nu}$ :

$$\partial_{[\mu} \left( \frac{1}{(\partial_\rho a \partial^\rho a)} \partial_{\nu]} a F_{\lambda\rho\sigma}^- \partial^\sigma a \right) = 0 \quad (30)$$

with the general solution:

$$F_{\mu\nu\lambda}^- \partial^\lambda a = \delta_2(\phi_\nu) [F_{\mu\nu\lambda}^- \partial^\lambda a] \quad (31)$$

which is equivalent to the self-duality condition due to the gauge invariance (29), which allows us to bring to zero the r.h.s of (31), and the self-duality equation follows. As mentioned above, this approach was used for the construction of the 5-brane action [10], and also the actions for chiral supergravities as in [13].

In the next Section we shall establish a connection between the two approaches, and in particular between expressions and symmetries of actions for chiral supergravities.

## 5 The Ansatz

Evidently actions (13) and (27) are connected by the ansatz for  $\theta$ :

$$\theta_{\mu\nu\lambda}^{++\rho\sigma\delta} = 6 \left( \frac{\partial_\mu a \partial^\rho a}{(\partial_\chi a \partial^{\chi a})} \delta_\nu^\sigma \delta_\lambda^\delta \right)^{++} = 6 (N_{\mu\nu\lambda}^{\rho\sigma\delta})^{++} \quad (32)$$

$$\theta_{\mu\nu\lambda}^{++\rho\sigma\delta} F_{\rho\sigma\delta}^- = 6 F_{\rho[\nu\lambda]}^- \left( \frac{\partial_{[\mu] a \partial^\rho a}}{(\partial_\sigma a \partial^{\sigma a})} - \frac{1}{6} \delta_{[\mu]}^\sigma \right) \quad (33)$$

or, equivalently, using (7) to (12) and (14) to (16)

$$\begin{aligned} \theta_{\mu\nu\lambda}^{++\rho\sigma\delta} &= 6 \left( \frac{\partial_{[\mu} a \partial^{\gamma} a}{(\partial_\chi a \partial^{\chi a})} - \frac{1}{6} \delta_{[\mu]}^\gamma \right) P_{\gamma\nu\lambda]}^{-\rho\sigma\delta} \\ &= 3 (P^+ N P^-)_{\mu\nu\lambda}^{\rho\sigma\delta} + 3 (P^+ N P^-)^{\rho\sigma\delta}_{\mu\nu\lambda} \end{aligned} \quad (34)$$

The ansatz, which we shall use to establish the connection between supergravities, differs from (34) in that all indices of  $\theta$  have to be flat ones, so that world indices of derivatives of the scalar field have to be transformed into flat ones by sixbeins. With this understanding, it can easily be checked term-by-term that Lagrangians of [13] can be obtained from that of [11] by this substitution of  $\theta$ . The question arises: What happens with the symmetry transformations and the algebra of symmetries? It is easy to check, that the first PST symmetry (28) is a particular case of the  $\alpha$ -symmetry (21), with the following ansatz for the parameter  $\alpha$ :

$$\alpha^\lambda(x) = \frac{\varphi(x) \partial^\lambda a}{(\partial_\chi a \partial^{\chi a})} \quad (35)$$

In particular, the entire transformation (22) follows from the shift of  $a$  in the ansatz (34). The same statement applies to the supersymmetry transformation. The susy transformations of [13] are in agreement with that of [11], particularly with (25), under the ansatz (34). Since in the PST formalism the susy transformation of the auxiliary scalar field  $a$  is zero, the whole expression (25) essentially originates from the transformation of the metric in the scalar product of ansatz (34), or from transformation of sixbeins in the same expression, needed for conversion of world indices of  $\partial_\mu a$  into flat ones of the tensor  $\theta$ . Next, there is no analogue for the second PST symmetry in

(29). This raises the question about the commutator of susy transformations (24), which according to [13] contain both PST symmetries. Remarkably, the expression for the  $\alpha$ -symmetry transformation in the commutator (24) splits into a sum of two PST symmetries with special parameters:

$$\begin{aligned}
\delta(\alpha)A_{\mu\nu} &= \frac{2(\alpha^\lambda\partial_\lambda a)}{(\partial_\chi a\partial^{\chi a})}F_{\mu\nu\rho}^-\partial^\rho a \\
&+ \partial_\mu a \left[ \frac{2F_{\nu\lambda\rho}^-\alpha^\lambda\partial^\rho a}{(\partial_\chi a\partial^{\chi a})} \right] - \partial_\nu a \left[ \frac{2F_{\mu\lambda\rho}^-\alpha^\lambda\partial^\rho a}{(\partial_\chi a\partial^{\chi a})} \right] \\
&= \delta_1(\alpha^\lambda\partial_\lambda a)A_{\mu\nu} + \delta_2\left(\frac{2F_{\gamma\lambda\rho}^-\alpha^\lambda\partial^\rho a}{(\partial_\chi a\partial^{\chi a})}\right)A_{\mu\nu}
\end{aligned} \tag{36}$$

Thus although the formalism [11] does not contain the second PST symmetry, the latter arises, after substitution of the ansatz, from the  $\alpha$ -symmetry transformation term in the commutator of the susy transformations. This last point completes our establishment of the connection between supergravity Lagrangians of [11] and those of [13] (as communicated earlier privately [14]).

## 6 Conclusions

Above we have established the connection between two formalisms, i.e. that of  $\alpha$ -symmetry, used about ten years ago for the construction of Lagrangians of supergravities with (anti)-self-dual tensors, and the recently invented PST formalism, which solves the same problem, and permits, in addition, the construction of the 5-brane Lagrangian. It appears, that Lagrangians of supergravities are connected through the ansatz, which connects the auxiliary field of both formalisms, i.e. the tensor  $\theta$  and the scalar  $a$ . The transformation of  $\alpha$ -symmetry terms into the PST symmetries, in the algebra of supersymmetry transformations, has been demonstrated. Also the agreement of rules for symmetries, particularly supersymmetries, has been demonstrated above.

Beyond the scope of this connection there remains the problem of the Lagrangian for the 5-brane, which so far has been established in Lorentz invariant form only in the PST formalism. Probably the latter can be constructed within the  $\alpha$ -symmetry formalism by considering more general expressions, satisfying the  $\alpha$ -symmetry requirement. It may be noted that  $\alpha$ -symmetry



strongly resembles reparametrization invariance, and the problem is something like the construction of actions with general coordinate invariance.

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