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AdS_3/CFT Correspondence, Poincaré Vacuum State and Greybody Factors in BTZ Black Holes

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Abstract

The greybody factors in BTZ black holes are evaluated from 2D CFT in the spirit of AdS_3/CFT correspondence. The initial state of black holes in the usual calculation of greybody factors by effective CFT is described as Poincaré vacuum state in 2D CFT. The normalization factor which cannot be fixed in the effective CFT without appealing to string theory is shown to be determined by the normalized bulk-to-boundary Green function. The relation among the greybody factors in different dimensional black holes is exhibited. Two kinds of $(h, \bar{h}) = (1, 1)$ operators which couple with the boundary value of massless scalar field are discussed.

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Recently it has been proposed [1] that string/M-theory on $\mathrm{AdS}_d \times M$ (where M is a proper compact space) is dual to a conformal field theory (CFT) which lives on the boundary of the anti-de Sitter (AdS) space. Precise forms of the conjecture [1] of the AdS/CFT correspondence have been stated and investigated in refs. [2, 3]. The essence of this conjecture is that supposing the partition functions of the two theories are equal, the correlation functions in the CFT can be read off from the bulk theory and vice versa. One of the interesting examples is the duality between type IIB string theory on $\mathrm{AdS}_3 \times S^3 \times M_4$ and certain 2D CFT [4]-[8], since both sides of the theories are very well understood. It has been shown that there is an agreement between the Kaluza-Klein spectrum of supergravity and the spectrum of certain 2D CFT [4, 7].

On the other hand, much work on the absorption and Hawking radiation in 5D and 4D black holes has been done in the semiclassical analysis, D-brane picture, effective string model and effective 2D CFT [9]-[14]. Since 5D and 4D black holes are U-dual to BTZ black holes [15], the entropies of 5D and 4D black holes can be related to the entropy of the BTZ black holes [16]. Especially, in the large N limit [1], the geometries of 5D and 4D black holes are $\mathrm{AdS}_3 \times M$ (effectively near horizon region). So one expects to find the explanation of greybody factors of higher dimensional black holes in the context of BTZ black holes. In refs. [17, 18], the greybody factors in BTZ black holes have been discussed from the semiclassical point of view, where the wave equations are solved.

In the effective string calculation of greybody factors in 5D and 4D black holes, the interaction between the scalar fields in the bulk and effective string is described by [12]

$$S_{int} = \int d^2x \phi(t, x, \vec{x} = 0)\mathcal{O}(t, x), \tag{1}$$

where the integration is over the effective string worldsheet, $\vec{x}$ indicates its location in transverse space, and $\mathcal{O}(t, x)$ is some local conformal operator in 2D CFT, which takes the form

$$\mathcal{O}(t, x) = \mathcal{O}_+(x^+)\mathcal{O}_-(x^-), \tag{2}$$

where $x^\pm = t \pm x$ and $\mathcal{O}_+$ and $\mathcal{O}_-$ are primary fields of dimension h_L and h_R, respectively.

The OPE's of $\mathcal{O}_+$ and $\mathcal{O}_-$ with themselves are given by

$$\mathcal{O}_+(\bar{z})\mathcal{O}_+(\bar{w}) \quad = \quad \frac{C_{\mathcal{O}_+}}{(\bar{z} - \bar{w})^{2h_L}} + \text{less singular terms},$$

$$\mathcal{O}_-(z)\mathcal{O}_-(w) \;=\; \frac{C_{\mathcal{O}_-}}{(z-w)^{2h_R}} + \text{less singular terms}, \tag{3}$$

where $z = ix^-, \bar{z} = ix^+$ for real x and imaginary $t = -i\tau$. To compare with the macroscopic decay rate in 5D and 4D black holes, the initial state is usually thermally averaged, since the black hole corresponds to a thermal state. This means that one must take finite temperature two-point correlation functions. Thus the function $G(t,x)$ defined by

$$G(t,x) = \langle \mathcal{O}(-i\tau, x)\mathcal{O}(0,0)\rangle_{T_H}, \tag{4}$$

is usually chosen as

$$G(t,x) = \frac{C_{\mathcal{O}_+}C_{\mathcal{O}_-}}{i^{2h_L+2h_R}} \left(\frac{\pi T_L}{\sinh \pi T_L x^+}\right)^{2h_L} \left(\frac{\pi T_R}{\sinh \pi T_R x^-}\right)^{2h_R}. \tag{5}$$

Since x^+ and x^- are linear in $\bar{z}$ and z, respectively, there seems to be some disagreement between eqs. (5) and (3). In other words, it is not clear what kind of initial state is used to define (4). Then natural questions arise how to describe the initial state of black holes in 2D CFT, and how to explain the greybody factors in 5D and 4D black holes in terms of those in BTZ black holes. These are the problems we are going to examine.

In order to answer the above questions, we discuss the greybody factors in BTZ black holes from 2D CFT in the spirit of AdS$_3$/CFT correspondence. To get explicit description for (4) and (5), we calculate the two-point correlation functions in the BTZ coordinates by bulk-boundary correspondence [1-3,18-21], but here we should include all the coefficients in the calculation. Using two-point correlation functions in BTZ coordinates, we evaluate the greybody factors in BTZ black holes, and find that they agree with the known results even for the coefficients. This fact gives further evidence to the AdS$_3$ $\leftrightarrow$ CFT correspondence. In fact, the calculation heavily relies on Witten's conjecture [3]. The result obtained shows that the initial state of black holes can be described by Poincaré vacuum state in 2D CFT. The coordinate transformation between Poincaré coordinates (w_+, w_-) and BTZ coordinates (u_+, u_-) induces a mapping of the operator $\mathcal{O}(w_+, w_-)$ to $\mathcal{O}(u_+, u_-)$ by Bogoliubov transformation, and the operator $\mathcal{O}(u_+, u_-)$ sees the Poincaré vacuum state (which is a natural vacuum state for 2D CFT) as an excited mixed state.

After explaining the greybody factors in BTZ black holes, we find that the greybody factors in 5D and 4D black holes can be described by those in BTZ black holes in a unified way. This is because in the large N limit [1], the geometries of 5D and 4D black holes turn out to be BTZ $\times S^3 \times M_4$ and BTZ $\times S^2 \times M_5$, respectively, and the two-point correlation functions in 5D and 4D black holes can be related to those in BTZ black holes, which are constant multiples of those in BTZ case. The constant can be determined from the parent 10D supergravity theories [20, 13, 23]. It is known that the greybody factors in black holes only depend on the conformal dimension of the operator $\mathcal{O}(t, x)$, and are indifferent to the concrete form of the operator. In order to understand the relation between the physical degrees of freedom in 3D pure gravity and those of 2D CFT, we next consider an $(h, \bar{h}) = (1, 1)$ operator in $\mathcal{N} = (4, 4)$ super CFT (SCFT) based on a resolution of the orbifold $(T^4)^{Q_1Q_5}/S(Q_1Q_5)$, and the corresponding operator in 3D gravity obtained by quantizing BTZ black holes in external massless scalar field. By comparing two operators, we find that the contribution from different scalars x_A is smeared, and gravity cannot distinguish between different CFT states with the same expectation value for the operator $\mathcal{O}(u_+, u_-)$, which shows that gravity is just like thermodynamics but gauge theory is like statistical mechanics [5, 24].

Now let us consider two-point correlation functions of 2D CFT; in Poincaré coordinates many studies of correlation functions in boundary CFT have been done by the bulk-boundary correspondence [19, 20, 21]; in BTZ coordinates this was analysed in ref. [22]. To calculate the greybody factors in BTZ black holes, we need to keep all the coefficients in the two-point correlation functions, and then compare the greybody factors extracted from AdS$_3$/CFT correspondence with those obtained from the semiclassical analysis.

As a first simple exploration, we consider two-point correlation functions of the operator coupling to the boundary value of the massive scalar field. In terms of Poincaré coordinates, the AdS$_3$ metric is

$$ds^2 = \frac{l^2}{y^2}(dy^2 + dw_+ dw_-). \tag{6}$$

For simplicity, we choose $l = 1$ in the following discussions.

The Euclidean action of massive scalar field with mass m in AdS$_3$ space is

$$S(\phi) = \frac{1}{2}\int dy dw_+ dw_- \sqrt{g}\left(g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + m^2\phi^2\right), \tag{7}$$

which has solution behaving as $\phi(y, w_+, w_-) \to y^{2h_-}\phi_0(w_+, w_-)$ when $y \to 0$. The boundary value $\phi_0(w_+, w_-)$ has dimension $2h_-$ which couples to an operator $\mathcal{O}(w_+, w_-)$ of dimension $2h_+$ with the parameters $h_\pm$ defined by

$$h_\pm = \frac{1}{2}(1 \pm \sqrt{1+m^2}). \tag{8}$$

The normalized bulk-to-boundary Green function in Poincaré coordinates is [20]

$$K_P(y, w_+, w_-; w'_+, w'_-) = \frac{\Gamma(2h_+)}{\pi\Gamma(2h_+ - 1)} \left[\frac{y}{y^2 + (w_+ - w'_+)(w_- - w'_-)}\right]^{2h_+}, \tag{9}$$

which has singular behavior when $y \to 0$ as [3, 20]

$$y^{-2h_-} K_P(y, w_+, w_-; w'_+, w'_-) \to \delta(w_+ - w'_+)\delta(w_- - w'_-). \tag{10}$$

Here we should mention that the behavior (10) determines the normalization coefficient in (9). The solution $\phi(y, w_+, w_-)$ can be expressed as

$$\phi(y, w_+, w_-) = \int dw'_+ dw'_- K_P(y, w_+, w_-; w'_+, w'_-)\phi_0(w'_+, w'_-), \tag{11}$$

where $\phi_0(w_+, w_-)$ is the boundary value of the bulk field $\phi(y, w_+, w_-)$.

The metric of BTZ black holes is [15]

$$ds^2 = -\frac{(r^2 - r_+^2)(r^2 - r_-^2)}{r^2} dt^2 + \frac{r^2}{(r^2 - r_+^2)(r^2 - r_-^2)} dr^2 + r^2 \left(d\phi - \frac{r_+ r_-}{r^2} dt\right)^2, \tag{12}$$

with periodic identification $\phi \sim \phi + 2\pi$, where we have chosen $l = 1$. The mass and angular momentum of BTZ black holes are defined as

$$M = r_+^2 + r_-^2, \quad J = 2r_+ r_-. \tag{13}$$

It can be shown that the metric of BTZ black holes can be transformed to that of AdS_3 locally by [4]

$$\begin{aligned} w_\pm &= \left(\frac{r^2 - r_+^2}{r^2 - r_-^2}\right)^{\frac{1}{2}} e^{2\pi T_\pm u_\pm}, \\ y &= \left(\frac{r_+^2 - r_-^2}{r^2 - r_-^2}\right)^{\frac{1}{2}} e^{\pi(T_+ u_+ + T_- u_-)}, \end{aligned} \tag{14}$$

with

$$T_\pm = \frac{r_+ \mp r_-}{2\pi}, \quad u_\pm = \phi \pm t. \tag{15}$$

In the region $r >> r_\pm$ in the BTZ coordinates, eqs. (14) can be approximated as

$$w_\pm = e^{2\pi T_\pm u_\pm}, \quad y = \left(\frac{r_+^2 - r_-^2}{r^2}\right)^{\frac{1}{2}} e^{\pi(T_+ u_+ + T_- u_-)}. \tag{16}$$

Since the boundary field $\phi_0(w_+, w_-)$ has conformal dimension (h_-, h_-), it is easy to see that near the boundary $y \to 0 (r \to \infty)$, the bulk field $\phi(r, u_+, u_-)$ in BTZ coordinates behaves as

$$\phi(r, u_+, u_-) \sim (2\pi T_+)^{-h_-}(2\pi T_-)^{-h_-}(r_+^2 - r_-^2)^{h_-} r^{-2h_-} \phi_0(u_+, u_-). \tag{17}$$

By use of the conformal dimensions of $\phi_0(w_+, w_-)$ and the relations (16), eq. (11) in the region $r >> r_\pm$ is cast into

$$\phi(r, u_+, u_-) = \int du'_+ du'_- K_B(r, u_+, u_-; u'_+, u'_-)\phi_0(u'_+, u'_-), \tag{18}$$

with

$$\begin{aligned} K_B(r, u_+, u_-; u'_+, u'_-) &= \frac{2h_+ - 1}{\pi}\left(\frac{r_+^2 - r_-^2}{r^2}\right)^{h_+} (2\pi T_+)^{-h_+}(2\pi T_-)^{-h_+} \\ &\times \left\{\frac{\pi^2 T_+ T_-}{\frac{r_+^2 - r_-^2}{4r^2} e^{\pi[T_+(u_+ - u'_+) + T_-(u_- - u'_-)]} + \sinh \pi T_+(u_+ - u'_+) \sinh \pi T_-(u_- - u'_-)}\right\}^{2h_+} \end{aligned} \tag{19}$$

In the derivation of (19), we keep all the coefficients in the calculation, and use the normalized bulk-to-boundary Green function.

According to AdS$_3$/CFT correspondence, the relation between string theory in the bulk and field theory on the boundary is [3]

$$e^{-S_{eff}(\phi)} = \langle e^{\int_B \phi_0 \mathcal{O}} \rangle_{CFT}. \tag{20}$$

Since ϕ is the solution to equations of motion, the bulk contribution to $S_{eff}(\phi)$ is zero, and a boundary term contributes to it:

$$S_{eff} = \lim_{r \to \infty} \frac{1}{2} \int du_+ du_- \sqrt{g} g^{rr} \phi \partial_r \phi. \tag{21}$$

Combining (12) and (17)-(19), we find

$$\begin{aligned} S_{eff} &= -\frac{h_+(2h_+ - 1)}{\pi}(r_+^2 - r_-^2)(2\pi T_+)^{-1}(2\pi T_-)^{-1} \\ &\times \int du_+ du_- du'_+ du'_- \phi_0(u_+, u_-) \left(\frac{\pi T_+}{\sinh \pi T_+(u_+ - u'_+)}\right)^{2h_+} \\ &\times \left(\frac{\pi T_-}{\sinh \pi T_-(u_- - u'_-)}\right)^{2h_+} \phi_0(u'_+, u'_-). \end{aligned} \tag{22}$$

From (20) and (22), one has

$$\begin{aligned} G(t,\phi) &= \langle \mathcal{O}(u_+,u_-)\mathcal{O}(0,0)\rangle \\ &= \frac{2h_+(2h_+-1)}{\pi}\left(\frac{\pi T_+}{\sinh \pi T_+ u_+}\right)^{2h_+}\left(\frac{\pi T_-}{\sinh \pi T_- u_-}\right)^{2h_+}, \end{aligned} \tag{23}$$

where we have used (15) to simplify the expression.[1] However, due to the periodic identification $\phi \sim \phi + 2\pi$, the above expression for $G(t,\phi)$ should be modified by the method of images as [22]

$$\begin{aligned} G_T(t,\phi) &= \langle \mathcal{O}(u_+,u_-)\mathcal{O}(0,0)\rangle \\ &= \frac{2h_+(2h_+-1)}{\pi}\sum_{n=-\infty}^{\infty}\left(\frac{\pi T_+}{\sinh \pi T_+(\phi+t+2n\pi)}\right)^{2h_+}\left(\frac{\pi T_-}{\sinh \pi T_-(\phi-t+2n\pi)}\right)^{2h_+} \end{aligned} \tag{24}$$

The sum over $n \neq 0$ in (24) comes from the twisted sectors of operator $\mathcal{O}(u_+,u_-)$ in the orbifold procedure $u_\pm \sim u_\pm + 2n\pi$ for the BTZ black holes [4]. The greybody factors in BTZ black holes are given by [12, 13]

$$\begin{aligned} \sigma_{abs} &= \frac{\pi}{\omega}\int dt \int_0^{2\pi} d\phi e^{ip\cdot x}[G_T(t-i\epsilon,\phi) - G_T(t+i\epsilon,\phi)] \\ &= \frac{\pi}{\omega}\int dt \int_{-\infty}^{\infty} d\phi e^{ip\cdot x}[G(t-i\epsilon,\phi) - G(t+i\epsilon,\phi)] \\ &= \frac{2h_+(2h_+-1)(2\pi T_+ l)^{2h_+-1}(2\pi T_- l)^{2h_+-1}\sinh\left(\frac{\omega}{2T_H}\right)}{\omega\Gamma^2(2h_+)} \\ &\quad \times \left|\Gamma\left(h_+ + i\frac{\omega}{4\pi T_+}\right)\Gamma\left(h_+ + i\frac{\omega}{4\pi T_-}\right)\right|^2, \end{aligned} \tag{25}$$

where the infinite sum in $G_T(t,\phi)$ has changed the original integral region $0 \le \phi \le 2\pi$ into $-\infty \le \phi \le \infty$ and the parameter l has been switched on. The Hawking temperature T_H is defined by

$$\frac{2}{T_H} = \frac{1}{T_+} + \frac{1}{T_-}. \tag{26}$$

Here we should point out that in the usual derivation of greybody factors in 5D and 4D black holes [10, 12], the periodicity along the spatial direction ϕ is ignored, because it is assumed that the length of effective string is large compared to the typical wavelength

[1] According to ref. [20], the Ward identities suggest that the factor h_+ in eq. (23) may be modified to $2h_+ - 1$ due to singular nature of the two-point correlation functions. This affects the following expressions, but the results for massless particles remain the same.

of the particle, i.e., 2π is much larger than $\frac{1}{T_H}$. However, in our method the same result can be obtained without the above assumption due to the good behavior of $G_T(t, \phi)$.

In the $m^2l^2 \to 0$ limit ($h_+ = 1$), the decay rate for massless scalar field is

$$\begin{aligned} \Gamma &= \frac{\sigma_{abs}(h_+ = 1)}{e^{\omega/T_H} - 1} \\ &= \frac{\omega\pi^2 l^2}{(e^{\omega/2T_+} - 1)(e^{\omega/2T_-} - 1)}, \end{aligned} \tag{27}$$

which is consistent with semiclassical gravity calculations in [17, 18]. We note that there is a minor difference between (25) and that obtained in ref. [18] with $h_L = h_R = h_+$. In eq. (25) there is an extra factor h_+, however, when $h_+ = 1$, both coincide.

The agreement of greybody factors in BTZ black holes obtained from AdS$_3$/CFT correspondence with those from semiclassical gravity calculations indicates that the usual effective string theory is nothing but the boundary 2D CFT of AdS$_3$ space. Let us recall that the two-point correlation functions in Poincaré and BTZ coordinates can be written as

$$\begin{aligned} \langle \mathcal{O}(w_+, w_-)\mathcal{O}(w'_+, w'_-)\rangle &\sim \frac{1}{(w_+ - w'_+)^{2h_+}(w_- - w'_-)^{2h_+}}, \\ \langle \mathcal{O}(u_+, u_-)\mathcal{O}(u'_+, u'_-)\rangle &\sim \frac{2h_+(2h_+ - 1)}{\pi}\left[\frac{\pi T_+}{\sinh \pi T_+(u_+ - u'_+)}\right]^{2h_+}\left[\frac{\pi T_-}{\sinh \pi T_-(u_- - u'_-)}\right]^{2h_+} \end{aligned} \tag{28}$$

In (28), the Poincaré coordinates $w_\pm$ are related to BTZ coordinates $u_\pm$ by an exponential transformation (16) in the region $r >> r_\pm$. Comparing (28) with eqs. (2)-(5), we find that $\bar{z}$ and z are not linear in x^+ and x^-, but rather they should be related by an exponential transformation. The result obtained also shows that the initial state of BTZ black holes can be described in 2D CFT by Poincaré vacuum state. The operators $\mathcal{O}_+(w_+)$ and $\mathcal{O}_-(w_-)$ in Poincaré coordinates satisfy the OPE in eq. (3). However, the nonlinear coordinate transformation (16) introduces a mapping of the original operator $\mathcal{O}(w_+, w_-)$ in Poincaré coordinates to the new one $\mathcal{O}(u_+, u_-)$ in BTZ coordinates, and this induces the Bogoliubov transformation on the operators. The operator $\mathcal{O}(u_+, u_-)$ see Poincaré vacuum state as an excited mixed state; that is, they see the Poincaré vacuum state as thermal bath of excitations in BTZ modes [4, 22]. The usual procedure to thermally average the initial state of black holes (or scalar particles) in the calculation of greybody factors is just to measure Poincaré vacuum state by the operator $\mathcal{O}(u_+, u_-)$ in BTZ coordinates, which was vague in the former treatment of greybody factors by effective string model in 5D and 4D black holes.

Having understood the greybody factors in BTZ black holes, let us now discuss the greybody factors in 5D and 4D black holes in the light of the fact that in the large N limit, the geometries of 5D and 4D black holes are BTZ $\times S^3 \times M_4$ and BTZ $\times S^2 \times M_5$, respectively [4, 7, 13]. For example, consider near-horizon AdS_3 structure of 5D black holes ("boosted" D1/D5 configuration) in the large N limit [1]. Its metric is [4, 18]

$$ds^2 = \frac{r^2}{R^2}(-dt^2 + dx^2) + \frac{r_0^2}{R^2}(\cosh\sigma dt + \sinh\sigma dx)^2 + \frac{R^2}{r^2 - r_0^2}dr^2 + R^2 d\Omega_3^2 + \frac{r_1^2}{R^2}\sum_{i=1}^{4} dx_i^2, \quad (29)$$

where r_0 is the extremality parameter, r_1, r_5 and $r_0 \sinh\sigma$ are related to the charges of D1-brane (Q_1), D5-brane (Q_5) and momentum in 5D black holes, and $R^2 = r_1 r_5$. The (t, x, r) part of the metric (29) is the metric of BTZ black holes. The coordinates (t, x) in (29) can be used to construct BTZ coordinates $u_\pm$, from which the Poincaré coordinates $w_\pm$ can be introduced, and 2D CFT lives on the asymptotic boundary $r \to \infty$ [4].

In the background (29), two-point correlation function is modified by a factor η_{5D} [20, 13, 23]

$$\langle \mathcal{O}(w_+, w_-)\mathcal{O}(w'_+, w'_-)\rangle = \eta_{5D}\frac{2h_+(2h_+ - 1)}{\pi}\frac{1}{(w_+ - w'_+)^{2h_+}(w_- - w'_-)^{2h_+}}, \quad (30)$$

where the constant η_{5D} can be completely determined from the parent 10D supergravity theory, that is, from the geometry of (29) by the procedure in refs. [20, 4, 13, 23]. Following the discussion in BTZ black holes, it is easy to see that the greybody factors in 5D black holes is $\sigma_{abs}^{5D} = \eta_{5D}\sigma_{abs}^{BTZ}$. Similarly this conclusion is also true for 4D black holes. This indicates that the greybody factors in 5D and 4D black holes have their own origin in BTZ black holes, and the dynamical information of 5D and 4D black holes are encoded in BTZ black holes. Thus the boundary dynamics of BTZ black holes, which is controlled by 2D CFT, looks like hologram and constrains the essential information of 5D and 4D black holes.

As we have seen, the conformal dimension of operator $\mathcal{O}(w_+, w_-)$ dominates the greybody factors, and one need not determine the explicit form of the operator $\mathcal{O}(u_+, u_-)$. Now let us consider the explicit form for the $(h, \bar{h}) = (1, 1)$ operator. For D1/D5 system in type IIB string theory compactified on T^4, the $\mathcal{N} = (4, 4)$ 2D SCFT can be described by the resolution of the orbifold $(T^4)^{Q_1Q_5}/S(Q_1Q_5)$ [25]. By AdS/CFT correspondence, the (1,1) operator can be determined from the symmetries [14]

$$\mathcal{O}_{ij} = \partial x^i_A \bar{\partial} x^j_A, \tag{31}$$

where x^i_A are the scalar fields in the SCFT under consideration, $A = 1, 2, \cdots, Q_1Q_5$ and i is the vector index of $SO(4)$, the local Lorentz group of T^4. The other possible forms for $(h, \bar{h}) = (1,1)$ operator can be excluded by the symmetries in AdS/CFT correspondence [14]. The interaction between $\mathcal{O}_{ij}$ and minimal scalars h_{ij} (whose origin is the traceless symmetric deformations of the 4-torus in type IIB string compactified on T^4) is given by

$$S_{int} = \int d^2 z h_{ij} \partial x^i_A \bar{\partial} x^j_A. \tag{32}$$

On the other hand, the (1,1) operator can be introduced by quantizing BTZ black holes in 3D pure gravity. In ref. [26], it has been shown that the gauge potentials can be parametrized by

$$\begin{aligned} A_\phi &= \begin{pmatrix} a^3(u_+) & e^{-\rho} a^+(u_+) \\ e^\rho a^-(u_+) & -a^3(u_+) \end{pmatrix}, \\ \tilde{A}_\phi &= -\begin{pmatrix} \tilde{a}^3(u_-) & e^{-\rho} \tilde{a}^+(u_-) \\ e^\rho \tilde{a}^-(u_-) & -\tilde{a}^3(u_-) \end{pmatrix}, \end{aligned} \tag{33}$$

and the asymptotic metric of BTZ black holes takes the form

$$ds^2 = l^2 d\rho^2 - l^2 e^{2\rho} a^-(u_+) \tilde{a}^+(u_-) du_+ du_- + \cdots , \tag{34}$$

where the irrelevant subleading terms at large ρ have been omitted. Then the action (7) is transformed into

$$S_{eff} = \int du_+ du_- \mathcal{O}_{gravity}(u_+, u_-) \sin(\omega t - n\phi), \tag{35}$$

where

$$\mathcal{O}_{gravity}(u_+, u_-) = a^-(u_+) \tilde{a}^+(u_-), \tag{36}$$

and the classical solution with its asymptotic form

$$\phi(\rho, t, \phi) = (1 - ie^{-2\rho}) e^{i(\omega_+ t - n_+ \phi)} + (1 + ie^{-2\rho}) e^{i(\omega_- t - n_- \phi)}, \tag{37}$$

has been exploited to get (35) with $\omega = \omega_+ - \omega_-$ and $n = n_+ - n_-$. Comparing (36) with (31), one is led to the identification

$$\partial x_A \bar{\partial} x_A \leftrightarrow a^-(u_+)\tilde{a}^+(u_-), \tag{38}$$

which shows that the contribution to $\mathcal{O}(u_+, u_-)$ from different x_A^i is smeared as seen by the operator $\mathcal{O}_{gravity}(u_+, u_-)$. Namely 3D pure gravity cannot distinguish between different CFT states with the same expectation value for the operator $\mathcal{O}(u_+, u_-)$. From these observations, one concludes that 3D gravity is a kind of thermodynamics but gauge theory is statistical mechanics [24].

In the above discussion, we have only considered the greybody factors induced by massive scalar fields. It is highly nontrivial to check whether the above identifications for the initial state of black holes in 2D CFT hold valid also for the spinor field case in the AdS/CFT correspondence.

As we have seen, the $(h, \bar{h}) = (1, 1)$ operator $\mathcal{O}(u_+, u_-)$ can be easily obtained by quantizing BTZ black holes in 3D gravity, however, it is unclear whether we can get operators $\mathcal{O}(u_+, u_-)$ of higher conformal dimensions by quantization of 3D gravity. If not, it is worth discussing whether and how they can be induced in the context of six-dimensional supergravity on $\mathrm{AdS}_3 \times S^3$, since the Kaluza-Klein spectrum of 6D supergravity truncated by 'stringy exclusion principle' matches the spectrum of 2D SCFT [4, 7].

Recently it has been argued that the isometry group $SL(2, R)$ of quantum gravity on AdS_2 can be enlarged to the full infinite-dimensional $1 + 1$ conformal group, and the mapping $\mathrm{AdS}_3 \to \mathrm{AdS}_2$ has been found [27]. It would be also interesting to see whether it is possible to find the origin of greybody factors in AdS_2 context as well.

We hope to return to these issues in near future.

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