Light-Cone Formulation of Super D2-Brane

R. Manvelyan[1 * †], **A. Melikyan**[2 ‡]
R.Mkrtchyan[2 §], **H.J.W. Müller–Kirsten**[1 ¶]

[1]*Department of Physics, University of Kaiserslautern, P. O. Box 3049, D 67653 Kaiserslautern, Germany*

[2]*Theoretical Physics Department, Yerevan Physics Institute, Alikhanian Br. St.2, Yerevan, 375036 Armenia*

Abstract

The light–cone Hamiltonian approach is applied to the super D2-brane, and the equivalent area–preserving and U(1) gauge–invariant effective Lagrangian, which is quadratic in the U(1) gauge field, is derived. The latter is recognised to be that of the three–dimensional U(1) gauge theory, interacting with matter supermultiplets, in a special external induced supergravity metric and the gravitino field, depending on matter fields. The duality between this theory and 11d supermembrane theory is demonstrated in the light–cone gauge.

[*]E-mail: manvel@physik.uni-kl.de
[†]Alexander von Humboldt Fellow,
On leave from Yerevan Physics Istitute
[‡]E-mail: arsen@moon.yerphi.am,
[§]E-mail: mrl@amsun.yerphi.am
[¶]E-mail: mueller1@physik.uni-kl.de

1 Introduction

In recent years significant progress was made in the theory of higher–dimensional extended supersymmetric objects. Extended objects play a central role in our understanding of non-perturbative aspects of superstring and supergravity theories. Moreover, the two-brane plays a crucial role in M-theory[1]. M-theory is intrinsically eleven-dimensional, and includes, in the spectrum of excitations, eleven-dimensional supermembrane theory. Besides the "usual" p-branes, with actions, described by scalars (brane coordinates in space-time) and fermions (superpartners of scalars), there exists a new class of extended objects, the so-called D-branes [2]. The D-branes contain in their spectrum a vector field (or, in the case of the eleven-dimensional 5-brane, a second rank self-dual tensor field). The main feature of the D-brane is that open superstrings can end on it. This means that open strings with Dirichlet boundary conditions describe the dynamics of the corresponding D-brane [2] connected by duality transformations with ordinary closed string backgrounds. The incorporation of D-branes in the superstring picture leads to a deeper understanding of the theory of solitonic states in non-perturbative string theory and of various aspects of string/M-theory dualities.

In ref. [3] we investigated the light-cone gauge for the bosonic part of the action of the D2-brane. The main result of that paper is that the corresponding gauged light-cone action for the Dirac-Born-Infeld Lagrangian can be rewritten as a three-dimensional Maxwell theory with matter field in the specific curved induced metric:

$$G_{\mu\nu} = \begin{pmatrix} -g + \xi^i(\omega)\, g_{ij}\, \xi^j(\omega) & \xi^k(\omega)\, g_{kj} \\ \xi^k(\omega)\, g_{ki} & g_{ij} \end{pmatrix} \tag{1}$$

where $\omega(\tau, \sigma_i)$ is the area-preserving gauge field, $\mu, \nu = 0, 1, 2$, and

$$\begin{aligned} g_{ij} &= \partial_i X^a \partial_j X^a, \quad \xi^i(\omega) = \varepsilon^{ki} \partial_k \omega \\ g &= \det g_{ij}, \quad a = 1,..8; \quad i,j = 1,2 \end{aligned}$$

The main goal of the present paper is to extend our previous investigation to the supersymmetric case. This leads to the interesting three dimensional structure of the effective light-cone action for the super D2-brane with the covariant interaction of abelian gauge and matter supermultiplets with induced supergravity multiplet formed by (1) and the induced gravitino field

ψ_μ. This action is closely related to extended supergravity in d=3, and to ordinary eleven-dimensional supermembrane theory in the light-cone gauge [4]. The latter allows the duality transformation to be established between these two extended objects[1] in the light-cone gauge in terms of our induced metric (1). In this article we prove that the effective action obtained from the Hamiltonian formulation for the super D2-brane in the light-cone gauge, exactly coincides with the action obtained from the supermembrane action in d=11 after duality transformation with our induced metric.

2 Hamiltonian formulation of super D2-brane and gauged Lagrangian

The light-cone formulation of the super-membrane obtained in [4] is closely connected with the matrix model representation of M-theory [5]. The corresponding area-preserving Lagrangian, from which we can obtain the matrix model by replacing Lie brackets by commutators, is [4]:

$$\begin{aligned} L_M &= \frac{1}{2}(D_0 X^{\dot a})^2 - \frac{1}{4}\left\{X^{\dot a}, X^{\dot b}\right\}\left\{X^{\dot a}, X^{\dot b}\right\} \\ &\quad -iS^T D_0 S - iS^T \gamma^{\dot a}\left\{X^{\dot a}, S\right\} \end{aligned} \tag{2}$$

where $\dot a, \dot b = 1, 2, ...9$, and S is the $SO(9)$ spinor coordinate, and $D_0 = \partial_0 + \{\omega, ...\}$ is a covariant area-preserving derivative with gauge field $\omega(\tau, \sigma_1, \sigma_2)$ and Lie bracket

$$\{X, Y\} = \varepsilon^{ij}\partial_i X \partial_j Y\,, \qquad i, j, .. = 1, 2. \tag{3}$$

The Lagrangian (2) can be interpreted as that of a 10-dimensional super-Yang-Mills theory (if we start from an 11-dimensional target space for the membrane) reduced to one dimension.

Here we investigate the light-cone formulation of the 10-dimensional super D-membrane described by the following supersymmetric Dirac-Born-Infeld

[1] The connection between the D2-brane in 10d and the membrane in 11d was first observed by M. J. Duff and J. X. Lu [6]. Schmidhuber [7] and Townsend [9] established this connection in both directions.

(DBI) Lagrangian [8],[7] :

$$S = -\int d^3\sigma \sqrt{-\det\left(\Sigma_{\mu\nu} + \mathcal{F}_{\mu\nu}\right)} - \int (C_3 + C_1 \wedge \mathcal{F}). \tag{4}$$

Here

$$\begin{aligned}
\Sigma_{\mu\nu} &= \Pi_\mu^M \Pi_\nu^M, \quad \Pi_\mu^M = \partial_\mu X^M - i\bar{\theta}\Gamma^M \partial_\mu \theta, \\
\mathcal{F}_{\mu\nu} &= F_{\mu\nu} - b_{\mu\nu} = \partial_\mu A_\nu - i\bar{\theta}\Gamma_{11}\Gamma^M \partial_\mu \theta \left(\partial_\nu X^M - i\frac{1}{2}\bar{\theta}\Gamma^M \partial_\nu \theta\right) - (\mu \leftrightarrow \nu), \\
M, N &= 0, 1, ..9; \quad \mu, \nu = 0, 1, 2
\end{aligned}$$

and RR-forms C_3 and C_1 are determined by the condition [8]

$$d(C_3 + C_1 \wedge \mathcal{F}) = d\bar{\theta}(\frac{1}{2}\psi^2 + \mathcal{F}\Gamma_{11})d\theta, \tag{5}$$

with $\psi \equiv \Gamma^M \Pi^M$.

We shall construct the analogue of the area-preserving supermembrane action (2) for the DBI case,which will be the supersymmetrization of our previous result [3].

We start with the Hamiltonian formulation of action (4) following the scheme of [4] and [3] after first imposing the light-cone gauge condition:

$$X^+(\tau, \sigma_i) = X^+(0,0) + \tau, \qquad X^\pm = \sqrt{\frac{1}{2}}\left(X^{10} \pm X^0\right) \tag{6}$$

$$\Gamma^+\theta = 0, \qquad \Gamma^\pm = \sqrt{\frac{1}{2}}\left(\Gamma^{10} \pm \Gamma^0\right). \tag{7}$$

After some tedious but straightforward calculations we can write down the corresponding final Hamiltonian density and the residual area preserving and Gauss-law (corresponding to $U(1)$ gauge invariance) constraints:

$$\begin{aligned}
H &= \frac{1}{2}\left[P^a P^a + P_A^i P_A^j g_{ij} + \det(g_{ij} + F_{ij})\right] \\
&\quad - P_A^i \bar{P}_\theta \Gamma_{11} \partial_i \theta + \bar{P}_\theta \Gamma^a \left\{X^a, \theta\right\}, && (8) \\
\Phi &= \varepsilon^{ij}\partial_i (P^a \partial_j X^a + P_A^k F_{jk} + \bar{P}_\theta \partial_j \theta) \approx 0, && (9) \\
\Upsilon &= \partial_i P_A^i \approx 0, && (10) \\
\bar{P}_\theta &+ i\bar{\theta}\Gamma^- \approx 0, && (11) \\
a &= 1, 2 ... 8; \; i, j, k = 1, 2
\end{aligned}$$

Here $P^a, P_A^i, \bar{P}_\theta$ are momenta corresponding to fields X^a, A_i and θ. Note that here, as in the bosonic case [3], we first fix the gauge $A_0 = 0$ and then put $P^+ = 1$ and correctly reexpress the X^- coordinate through the transverse ones :

$$\partial_i X^- = -(P^a \partial_i X^a + P_A^j F_{ij} + \bar{P}_\theta \partial_i \theta) \tag{12}$$

using the additional gauge condition:

$$\partial_0 X^a \partial_i X^a + \partial_i X^- - i\bar{\theta}\Gamma^- \partial_i \theta + (\partial_0 A_k - i\bar{\theta}\Gamma^- \Gamma_{11} \partial_k \theta) g^{kj} F_{ij} = 0 \tag{13}$$

Moreover, in the gauge (6) , (7) , (13) the expressions for the momenta be come very simple:

$$\begin{aligned} P^a &= \partial_0 X^a \\ P_A^i &= g^{ij}(\partial_0 A_j - i\bar{\theta}\gamma_- \Gamma_{11} \partial_j \theta) \\ \bar{P}_\theta &= -i\bar{\theta}\Gamma^- \end{aligned} \tag{14}$$

where

$$g^{ij} = \frac{\varepsilon^{ik}\varepsilon^{jl} g_{kl}}{g} \tag{15}$$

So, finally we can proceed to define the effective gauged Lagrangian containing fields X^M, θ, A_i and two gauge fields $\omega(\tau, \sigma_i)$ and $Q(\tau, \sigma_i)$ with the following properties:

1. The expressions (14) and (8) have to be derived as standard expressions of the canonical momenta and Hamiltonian for that Lagrangian in the gauge

$$\begin{aligned} \omega(\tau, \sigma_i) &= 0 \\ Q(\tau, \sigma_i) &= 0 \end{aligned} \tag{16}$$

2. The equations of motion for gauge fields $\omega(\tau, \sigma_i)$ and $Q(\tau, \sigma_i)$ have to coincide (in the gauge (16)) with corresponding constraints (9) and (10).

3. This Lagrangian has to be gauge invariant under the following gauge groups:

a) the group of area–preserving diffeomorphisms with generator corresponding to the constraint (9),

b) the group of $U(1)$ gauge transformations connected with constraint (10) .

The desired effective Lagrangian has the following form:

$$\begin{aligned} L &= \frac{1}{2}(D_0 X^a)^2 - \frac{1}{4}\left\{X^a, X^b\right\}\left\{X^a, X^b\right\} - i\bar{\theta}\Gamma^- D_0\theta - i\bar{\theta}\Gamma^-\Gamma^a\left\{X^a, \theta\right\} \\ &+ \frac{1}{2}g^{ij}(D_0 A_i - \partial_i Q - i\bar{\theta}\Gamma^-\Gamma_{11}\partial_i\theta)(D_0 A_j - \partial_j Q - i\bar{\theta}\Gamma^-\Gamma_{11}\partial_j\theta) \\ &- \frac{1}{2}F_{12}^2 \end{aligned} \tag{17}$$

where

$$\begin{aligned} D_0 X^a &= \partial_0 X^a - \varepsilon^{ij}\partial_i\omega\partial_j X^a \\ &= \partial_0 X^a - \{\omega, X^a\} = \partial_0 X^a - \pounds_{\xi(\omega)} X^a \\ D_0 A_i &= D_0 A_i = \partial_0 A_r - \varepsilon^{kj}\partial_i\partial_k\omega A_j - \varepsilon^{kj}\partial_k\omega\partial_j A_i \\ &= \partial_0 A_i - \pounds_{\xi(\omega)} A_i \\ D_0\theta &= \partial_0\theta - \varepsilon^{ij}\partial_i\omega\partial_j\theta \end{aligned} \tag{18}$$

Here $\pounds_{\xi(\omega)}$ is the Lie derivative in the direction of the divergenceless vector field $\xi^i(\omega) = \varepsilon^{ki}\partial_k\omega$. Lagrangian (17) satisfies all three conditions. The gauge transformations of the given fields are the following:

$$\begin{aligned} \delta_\varepsilon X^a &= \{\varepsilon, X^a\}, \quad \delta_\varepsilon\theta = \{\varepsilon, \theta\} \\ \delta_\varepsilon A_i &= \pounds_{\xi(\omega)} A_i, \\ \delta_\varepsilon Q &= \{\varepsilon, Q\}, \quad \delta_\varepsilon\omega = \partial_0\varepsilon + \{\varepsilon, \omega\}, \\ \delta_\alpha X^a &= 0, \quad \delta_\alpha A_i = \partial_i\alpha, \quad \delta_\alpha Q = \partial_0\alpha + \{\alpha, \omega\}, \\ \delta_\alpha\omega &= 0, \quad \delta_\alpha\theta = 0. \end{aligned} \tag{19}$$

It is easy to see that here as in the bosonic case, the $U(1)$ gauge transformations of Q do not commute with the area-preserving ones. This can be altered by a redefinition of the field Q:

$$A_0 = Q + \varepsilon^{ij}\partial_i\omega A_j \tag{20}$$

Here we introduce the new component A_0, which unlike Q transforms (under area-preserving diffeomorphisms) not as a scalar but as the zero component of a three-dimensional vector field :

$$\begin{aligned} \delta_\alpha A_0 &= \partial_0\alpha \\ \delta_\varepsilon A_0 &= \{\varepsilon, A_0\} + \partial_0\xi^i(\varepsilon) A_i \end{aligned} \tag{21}$$

Then we can solve the light–cone condition (7) for the fermionic coordinate and rewrite our fermionic coordinates and gamma matrices in terms of $SO(8)$ quantities :

$$\theta = 2^{-1/4}\begin{pmatrix} S \\ 0 \end{pmatrix}, \bar{\theta}\Gamma^{-} = 2^{1/4}\begin{pmatrix} S^T & 0 \end{pmatrix}, \tag{22}$$

$$\Gamma^a = \begin{pmatrix} \gamma^a & 0 \\ 0 & -\gamma^a \end{pmatrix}, \quad \Gamma_{11} = \begin{pmatrix} \gamma_9 & 0 \\ 0 & -\gamma_9 \end{pmatrix}. \tag{23}$$

With this the Lagrangian (17) can be rewritten in the following form:

$$\begin{aligned} L &= \frac{1}{2}(D_0X^a)^2 - \frac{1}{4}\left\{X^a, X^b\right\}\left\{X^a, X^b\right\} \\ &\quad -iS^T D_0 S - iS^T\gamma^a\left\{X^a, S\right\} \\ &\quad +\frac{1}{2}g^{ij}\tilde{F}_{0i}\tilde{F}_{0j} - \frac{1}{2}F_{12}^2 \end{aligned} \tag{24}$$

where

$$\begin{aligned} \tilde{F}_{0i} &= F_{0i} - \xi^k F_{ki} - iS^T\gamma_9\partial_i S \\ F_{0i} &= \partial_0 A_i - \partial_i A_0 \end{aligned}$$

Unlike the bosonic case, this Lagrangian is supersymmetric with the following field transformations:

$$\begin{aligned} \delta X^a &= i\varepsilon^T\gamma^a S, \\ \delta\omega &= i\varepsilon^T S, \\ \delta S^T &= \frac{1}{2}\varepsilon^T D_0 X^a\gamma_a + \frac{1}{2}\varepsilon^T g^{ij}\tilde{F}_{0i}\gamma_9\gamma_a\partial_j X^a + \frac{1}{4}\varepsilon^T\left\{X^a, X^b\right\}\gamma_{ab} - \frac{1}{2}\varepsilon^T F_{12}\gamma_9, \\ \delta A_i &= i\varepsilon^T\gamma_9\gamma_a S\partial_i X^a, \\ \delta A_0 &= i\varepsilon^T\gamma_9\gamma_a S\partial_0 X^a + i\delta S^T\gamma_9 S \end{aligned} \tag{25}$$

3 Induced 3d formulation and duality

We introduce the three dimensional induced metric $G_{\mu\nu}$ of (1) with the following properties:

$$G_{\mu\nu} = \begin{pmatrix} -g + \xi^i(\omega)\, g_{ij}\xi^j(\omega) & \xi^k(\omega)\, g_{kj} \\ \xi^k(\omega)\, g_{ki} & g_{ij} \end{pmatrix},$$

$$
\begin{aligned}
G^{\mu\nu} &= \begin{pmatrix} -1/g & \xi^j(\omega)/g \\ \xi^i(\omega)/g & g^{ij} - \xi^i(\omega)\xi^j(\omega)/g \end{pmatrix}, \\
g_{ij} &= \partial_i X^a \partial_j X^a,\ g = \det g_{ij}\,,\ \xi^i(\omega) = \varepsilon^{ki}\partial_k \omega \\
\det G_{\mu\nu} &= G,\ \sqrt{-G} = g, \qquad \sqrt{-G}G^{00} = -1
\end{aligned} \tag{26}
$$

We then construct the induced three dimensional curved Dirac matrices using the following definitions:

$$
\begin{aligned}
\gamma_\mu &= (\gamma_0, \gamma_i), \quad \gamma_i = \partial_i X^a \gamma^a, \\
\gamma_0 &= \gamma + \xi^i(\omega)\gamma_i, \quad \gamma = \frac{1}{2}\varepsilon^{ij}\gamma_i\gamma_j \\
\gamma^\mu &= G^{\mu\nu}\gamma_\nu = (-\frac{\gamma}{g}, \frac{\xi^i(\omega)\gamma}{g} + g^{ij}\gamma_j)
\end{aligned} \tag{27}
$$

These matrices satisfy a three-dimensional Clifford algebra with our induced metric (26):

$$
\{\gamma_\mu, \gamma_\nu\} = 2G_{\mu\nu} \tag{28}
$$

Finally we introduce the induced *gravitino field:*

$$
\begin{aligned}
\psi_\mu &= (\psi_0, \psi_i), \quad \psi_i = \partial_i S, \quad \psi_0 = \xi^i(\omega)\psi_i + \gamma_i \varepsilon^{ij}\psi_j \\
\psi^\mu &= G^{\mu\nu}\psi_\nu = (\psi^0, \psi^i) \\
\psi^0 &= -\frac{\gamma_i\varepsilon^{ij}\psi_j}{g}, \quad \psi^i = g^{ij}\psi_j + \frac{\xi^i(\omega)\gamma_k\varepsilon^{kl}\psi_l}{g}
\end{aligned} \tag{29}
$$

With this we can rewrite our light–cone action (24) in the following final covariant form on our induced metric:

$$
\begin{aligned}
L &= -\frac{1}{2}\sqrt{-G}G^{\mu\nu}\partial_\mu X^a \partial_\nu X^a + \frac{1}{2}\sqrt{-G} \\
&\quad + i\sqrt{-G}G^{\mu\nu}\bar{S}\gamma_\mu\partial_\nu S - \frac{1}{4}\sqrt{-G}G^{\mu\nu}G^{\sigma\lambda}\mathbf{F}_{\mu\sigma}\mathbf{F}_{\nu\lambda} \\
&\quad + \frac{i}{4}\varepsilon^{\mu\nu\lambda}\mathbf{F}_{\mu\nu}\bar{S}\gamma_9\psi_\lambda - \frac{1}{8}\sqrt{-G}G^{\mu\nu}\bar{S}\gamma_9\psi_\mu\bar{S}\gamma_9\psi_\nu,
\end{aligned} \tag{30}
$$

where

$$
\begin{aligned}
\mathbf{F}_{\mu\nu} &= F_{\mu\nu} + i\bar{S}\gamma_9\gamma_{[\mu}\psi_{\nu]} \\
\bar{S} &= S^T\gamma^0 = -S^T\frac{\gamma}{g}, \quad \partial_\mu = (\partial_0, \partial_i)
\end{aligned}
$$

and we used the following relations:

$$\begin{aligned}\gamma_j\varepsilon^{ji} &= \gamma\gamma_k g^{ki} = -g^{ik}\gamma_k\gamma, \quad \varepsilon_{ij} = g_{ik}g_{jl}\varepsilon^{kl}, \quad \varepsilon^{ik}\epsilon_{kj} = -\delta^i_j \\ \gamma_i\gamma_j &= g_{ij} + \epsilon_{ij}\gamma, \quad \epsilon_{ij} = \frac{\varepsilon_{ij}}{g}, \quad g_{ik}\varepsilon^{kj} = \gamma_i\gamma_k\varepsilon^{kj} + \gamma\delta^j_i.\end{aligned}$$

We have thus shown that the effective action for the light–cone 10d super D2-brane can be expressed in the form of a usual three-dimensional Abelian gauge field coupled to the matter fields X^a, S in the induced supergravity (26),(29) defined by the same matter fields - target space coordinates X^a, S. The expression for the zero component of the gravitino field (29) is in accordance with the supersymmetry transformation of our induced metric, which can be derived from (25) , (26) , (27) and (29):

$$\delta G_{\mu\nu} = i(\bar{\varepsilon}\gamma_\mu\psi_\nu + \bar{\varepsilon}\gamma_\nu\psi_\mu), \bar{\varepsilon} = \varepsilon^T \tag{31}$$

So this transformation looks like some global remainder of the initial local supersymmetry with the following choice of local supersymmetry parameter:

$$\begin{aligned}\varepsilon(\tau,\sigma_i) &= -\gamma(\tau,\sigma_i)\varepsilon \\ \gamma(\tau,\sigma_i) &= \frac{1}{2}\varepsilon^{ij}\partial_i X^a(\tau,\sigma_i)\partial_j X^b(\tau,\sigma_i)\frac{1}{2}\left[\gamma^a,\gamma^b\right]\end{aligned}$$

One should note that we have three 16×16 induced curved space gamma matrices, and our induced gravitino has 16 spinor components. We therefore have the $N = 8$, $d = 3$ supergravity multiplet [10] but our internal and spinor spaces have mixed in the one reducible representation.

Finally we note that our covariant Lagrangian (30) is quadratic in the Maxwell field and therefore, as in the bosonic case, it is possible to establish a direct duality relation with the M-theory membrane action (2).

For this reason we rewrite the membrane light–cone action (2) in the covariant form:

$$\begin{aligned}L_M &= -\frac{1}{2}\sqrt{-\tilde{G}}\tilde{G}^{\mu\nu}\partial_\mu X^{\dot{a}}\partial_\nu X^{\dot{a}} + \frac{1}{2}\sqrt{-\tilde{G}} \\ &+ i\sqrt{-\tilde{G}}\tilde{G}^{\mu\nu}\bar{S}\tilde{\gamma}_\mu\partial_\nu S, \quad \bar{S} = S^T\tilde{\gamma}^0\end{aligned} \tag{32}$$

Here the tilde metric and gamma matrices are induced from 9 dimensional transverse target space. Then we can separate in (32) the $X^{\dot{a}}$ into 8- dimensional target space coordinates X^a and X^9, and we can replace $\partial_\mu X^9$ by the independent vector field B_μ and add to L_M a metric independent topological term [2]:

$$S_M = \int L_M\left(X^a, B\right) d\tau d^2\sigma + \frac{1}{2}\int BdA \tag{33}$$

With this and using our definition of the induced gravitino field (29) and the following identity:

$$\varepsilon^{\mu\nu\lambda}\partial_\mu X^9 \bar{S}\gamma_9\gamma_\nu\psi_\lambda + \sqrt{-G}\partial_\mu X^9 \bar{S}\gamma_9\psi^\mu = -2\partial_i X^9 S^T\gamma_9\psi_j\varepsilon^{ij},$$

we can rewrite (33) in the following form:

$$\begin{aligned} S_M \quad &= \int d\tau d^2\sigma \left\{ -\frac{1}{2}\sqrt{-G}G^{\mu\nu}\partial_\mu X^a \partial_\nu X^a + \frac{1}{2}\sqrt{-G} \right. \\ &+ i\sqrt{-G}G^{\mu\nu}\left(\bar{S}\gamma_\mu\partial_\nu S + \frac{1}{2}B_\mu \bar{S}\gamma_9\psi_\nu\right) \\ &\left. -\frac{1}{2}\sqrt{-G}G^{\mu\nu}B_\mu B_\nu + \frac{1}{2}\varepsilon^{\mu\nu\lambda}B_\mu \mathbf{F}_{\nu\lambda} \right\} \end{aligned} \tag{34}$$

The equation of motion for A_μ leads to (32). But elimination of B_μ brings us back to our light-cone effective Lagrangian for the super D2-brane (30), which we obtained from the Hamiltonian formulation. As mentioned above, the connection between the 10d D2-brane and the 11d membrane was established and exploited in [6], [9], [7].

4 Conclusions

In the above the light–cone formalism has been developed for the 10–dimensional super D2-brane, and it was shown, that all corresponding equations of motion and constraints can be derived from the Lagrangian of a 3d Maxwell theory, interacting with matter fields, in a curved space-time with a special induced 3d supergravity multiplet. This theory is invariant with respect to the usual Abelian gauge transformations of gauge fields, and with respect to area-preserving diffeomorphisms and supersymmetry. So, we have shown, that as in the bosonic case [3], the non-linear super D2-brane Lagrangian can be replaced in the light–cone gauge, by a quadratic one over gauge fields, although the dependence on coordinates of the membrane remains highly non-linear.

[2] One may note that this topological term is supersymmetric because $\delta B_\mu \sim \partial_\mu(\bar{\varepsilon}\gamma^9 S)$.

The exact integration over gauge fields can now be carried out, at least formally, and the corresponding determinant has to be considered as an effective action for the super D2-brane, and can be expanded in terms of the curvature tensor constructed from the induced metric (26). Another direction in which to extend the results of this article is to explore the connection between the light–cone formulation with three dimensional extended supergravity and consideration of the super Dp-brane in the light–cone formulation for $p > 2$. The connection between the D-brane actions and supergravity in the corresponding dimensions is a long–standing and interesting problem. For p=2 the connetion between Green–Schwarz and Neveu–Schwarz actions is well–knwon and goes through the light–cone gauges. For higher dimensions some insight is considered in the recent works [11]. Present consideration shows, for the D2-brane, a possibility of introduction of the (induced) supergravity multiplet, with the (induced) susy transformations. This is a field for future investigations.

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