KL-TH/95/21

2D quantum dilaton gravitational Hamiltonian, boundary terms and new definition for total energy

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Abstract

The ADM and Bondi mass for the RST model have been first discussed from Hawking and Horowitz's argument. Since there is a nonlocal term in the RST model, the RST lagrangian has to be localized so that Hawking and Horowitz's proposal can be carried out. Expressing the localized RST action in terms of the ADM formulation, the RST Hamiltonian can be derived, meanwhile keeping track of all boundary terms. Then the total boundary terms can be taken as the total energy for the RST model. Our result shows that the previous expression for the ADM and Bondi mass actually needs to be modified at quantum level, but at classical level, our mass formula can be reduced to that given by Bilal and Kogan [5] and de Alwis [6]. It has been found that there is a new contribution to the ADM and Bondi mass from the RST boundary due to the existence of the hidden dynamical field. The ADM and Bondi mass with and without the RST boundary for the static and dynamical solutions have been discussed respectively in detail, and some new properties have been found. The thunderpop of the RST model has also been encountered in our new Bondi mass formula.

In recent years, the definition for the total energy in 2D dilaton gravity has attracted a lot of attention [1-7]. The formula in refs.[1-4] for the Arnowitt-Deser-Misner (ADM) mass of 2D dilaton gravity was found to be incomplete, and the origin of this incompleteness can be traced to the implicit assumption that $\delta\phi$ and $\delta\rho$ are $O(e^{-2\lambda\sigma})$, but they actually contain terms $O(e^{-\lambda\sigma})$ [5,6]. Bilal and Kogan [5] gave an improved expression for ADM mass by imposing some asymptotic conditions, and assumed no contribution to the ADM mass from $\sigma = -\infty$, but their mass formula did not contain the quantum corrections. Later, de Alwis [6] obtained the quantum corrected expressions for the ADM and Bondi mass using arguments given by Regge and Teitelboim [8], and found that there was a contribution to the ADM and Bondi mass from the point $\sigma \to -\infty$. However, the quantum versions of the original Callan-Giddings-Harvey-Strominger (CGHS) model [1], for instance, the Russo-Susskind-Thorlacious (RST) model [9], have been shown that there exists a hidden dynamical field [10-12], which was omitted in previous considerations of the semiclassical approach. So one may ask whether there is a contribution to the total energy from this hidden dynamical field.

On the other hand, in the above derivations of the 2D dilaton gravitational Hamiltonian, the boundary term has been ignored. This results in a Hamiltonian which is just a multiple of a constraint. Then one must add to this constraint appropriate boundary terms so that its variation is well defined [5,6]. Recently, Hawking and Horowitz [13] proposed to keep track of all surface terms in a general derivation of the gravitational Hamiltonian starting from the Einstein-Hilbert action. The resulted surface terms can be taken as the definition of the total energy even for spacetime that is not asymptotically flat [14]. Thus the boundary terms in H come directly from the boundary terms in the action, and do not need to be added "by hand". However, they just considered the case of the 4D Einstein-Hilbert action, in adopting 2D quantum dilaton gravity as a model for 4D gravity, it is important to know what features the two theories have in common. For example, is there a lower bound to the total energy for the RST model? and what role does the hidden dynamical field play in the total energy expression?

In the present paper, the ADM and Bondi mass for the RST model are first discussed using the argument given by Hawking and Horowitz [13]. Since there is a nonlocal term in the RST model, the RST lagrangian must first be localized so that Hawking and Horowitz's proposal can be carried out. For this purpose, the scalar field χ and the boundary term are introduced in order that the reformulated RST action is well-posed and local [10,11,15]. Expresssing the RST action in terms of the ADM formulation [15-19], the RST Hamiltonian can be derived, meanwhile keeping track of all boundary terms. Then the total boundary terms can be identified as the ADM or Bondi mass for the RST model. The result shows that the mass formula used in refs.[1-7] actually needs to be modified at quantum level. But at classical level, our mass formula can be reduced to that of refs.[5,6]. It has been shown that there is a new contribution to the total energy from the hidden dynamical field. In the absence of the RST boundary, that is, the left boundary of the spacetime is $\bar{\sigma} = -\infty$, the ADM mass for the static solutions is zero, whereas for the dynamical case with collapsing matter, it has been found that there is an infinite contribution to the ADM mass from the negative infinite end of the space, i.e., this theory describes black hole collapse in an infinite bath of radiation. The Bondi mass has also been discussed. However, in the case without the RST boundary, the Bondi mass is found to be finite. In the presence of the RST boundary, it has been shown that the ADM mass for the static solutions with $m_0 > 0$ (m_0 will be defined below) is zero, so we resolve the problem that was left in ref.[6]. In the dynamical case, a new contribution to the ADM mass from the RST boundary has been found, which is just the consequence that the hidden dynamical field affects the mass formula. When $\bar{\tau} \to -\infty$, the ADM mass goes to the mass of the collapsing matter. However, at an intermediate time the Bondi mass becomes negative, and is discontinuous across a certain null line, that is, the thunderpop of RST [9] has been encountered. In the region $\bar{\sigma}^- > \bar{\sigma}_s^-$ ($\bar{\sigma}_s^+, \bar{\sigma}_s^-$ are the points where the apparent horizon and the critical curve intersect), the black hole has decayed and the solution is taken to be linear dilaton vacuum (LDV), but the Bondi mass is found to be nonzero due to the existence of the hidden dynamical field, which reflects the fact that the spacetime is not globally flat. Our result also shows that with the new definition for the total energy, the ADM and Bondi mass have a lower bound in the presence of the RST boundary.

We now consider the RST model with the action [9]

$$
\begin{aligned}
S &= \frac{1}{4\pi}\int d^2\sigma\sqrt{-g}\left\{e^{-2\phi}\left[R+4(\nabla\phi)^2+4\lambda^2\right]-\frac{1}{2}\sum_{i=1}^{N}(\nabla f_i)^2\right. \\
&\quad \left. -\frac{\kappa}{4}\left(R\frac{1}{\nabla^2}R+2\phi R\right)\right\} \qquad (1)
\end{aligned}
$$

where $g_{\alpha\beta}$ is the metric on the 2D manifold M, R is its curvature scalar, ϕ is the dilaton field, and the $f_i, i=1,\cdots,N$, are N scalar matter fields.

According to refs.[10,11,15], one can introduce an independent scalar field χ to localize the conformal anomaly term, and add a boundary term to define the variational problem properly. Then Eq.(1) turns into [10-11]

$$
\begin{aligned}
S &= \frac{1}{4\pi}\int d^2\sigma\sqrt{-g}\left\{R\tilde{\chi}+4[(\nabla\phi)^2+\lambda^2]e^{-2\phi}-\frac{\kappa}{4}g^{\alpha\beta}\partial_\alpha\chi\partial_\beta\chi\right. \\
&\quad \left. -\frac{1}{2}\sum_{i=1}^{N}(\nabla f_i)^2\right\}-\frac{1}{2\pi}\int d\Sigma\sqrt{\pm h}K\tilde{\chi} \qquad (2)
\end{aligned}
$$

where $\tilde{\chi}=e^{-2\phi}-\frac{\kappa}{2}(\phi-\chi)$, h is the induced metric on the boundary of M, and K is the extrinsic curvature of ∂M. Following the ADM formulation, the metric can be parametrized as follows [15-19]:

$$g_{\alpha\beta}=e^{2\rho}\hat{g}_{\alpha\beta} \qquad (3)$$

$$\hat{g}_{\alpha\beta}=\begin{pmatrix} -\omega^2+\theta^2 & \theta \\ \theta & 1 \end{pmatrix} \qquad (4)$$

where $\omega(x)$ and $\theta(x)$ are lapse and shift functions respectively, and the conformal factor $e^{2\rho}$ has been factored out.

In terms of this parametrization, the action (2) can be written as

$$
\begin{aligned}
S &= \frac{1}{2}\int d^2\sigma\sqrt{\hat{g}}\left\{\hat{R}\tilde{\chi}+2\hat{g}^{\alpha\beta}\partial_\alpha\tilde{\chi}\partial_\beta\rho-2\hat{g}^{\alpha\beta}\partial_\alpha\phi\partial_\beta e^{-2\phi}\right. \\
&\quad \left. +4\lambda^2e^{2(\rho-\phi)}-\frac{\kappa}{4}\hat{g}^{\alpha\beta}\partial_\alpha\chi\partial_\beta\chi-\frac{1}{2}\sum_{i=1}^{N}\hat{g}^{\alpha\beta}\partial_\alpha f_i\partial_\beta f_i\right\} \\
&\quad -\int d\Sigma\sqrt{\pm h}K\tilde{\chi}-\int d^2\sigma\partial_\alpha\left[\sqrt{-\hat{g}}\hat{g}^{\alpha\beta}\tilde{\chi}\partial_\beta\rho\right] \qquad (5)
\end{aligned}
$$

where $\hat{R}$ is the curvature scalar for $\hat{g}_{\alpha\beta}$, and for simplicity, the factor $(2\pi)^{-1}$ in front of action (2) has been omitted (later it will be recovered). The last term in (5) is a surface term coming from the relation $\sqrt{-g}R = \sqrt{-\hat{g}}\hat{R} - 2\partial_\alpha(\sqrt{-\hat{g}}\hat{g}^{\alpha\beta}\partial_\beta\rho)$.

In order to obtain the RST Hamiltonian explicitly, we need a field redefinition to diagonalize the kinetic term of action (5), which is first given by

$$\begin{aligned}
\psi_0 &= \frac{1}{\sqrt{\kappa}}e^{-2\phi} - \frac{\sqrt{\kappa}}{2}\phi + \sqrt{\kappa}\rho \\
\psi_1 &= -\frac{\sqrt{\kappa}}{2}\chi + \sqrt{\kappa}\rho \\
\psi_2 &= \frac{1}{\sqrt{\kappa}}e^{-2\phi} + \frac{\sqrt{\kappa}}{2}\phi
\end{aligned} \tag{6}$$

where ψ_1 is the hidden dynamical field [10-12], which was omitted in the previous semi-classical consideration.

From (6), the action (5) can be written as

$$\begin{aligned}
S &= \int d^2\sigma \left\{ \frac{\sqrt{\kappa}}{\omega}(\dot{\psi}_0 - \dot{\psi}_1)\theta' + \frac{\sqrt{\kappa}}{\omega}(\psi_0' - \psi_1')(\omega\omega' - \theta\theta') \right. \\
&\quad \left. + \frac{1}{2}\omega\hat{g}^{\alpha\beta}\partial_\alpha\psi_\mu\partial_\beta\psi_\nu\eta^{\mu\nu} + 2\lambda^2\omega e^{\frac{2}{\sqrt{\kappa}}(\psi_0-\psi_2)} - \frac{1}{4}\omega\sum_{i=1}^{N}\hat{g}^{\alpha\beta}\partial_\alpha f_i\partial_\beta f_i \right\} \\
&\quad - \left\{ \int d\Sigma\sqrt{\pm h}K\tilde{\chi} + \int d^2\sigma\partial_\alpha\left[\sqrt{-\hat{g}}\hat{g}^{\alpha\beta}\tilde{\chi}\partial_\beta\rho\right] \right. \\
&\quad \left. + \int d\tau\left[\frac{\tilde{\chi}}{\omega}(\omega\omega' - \theta\theta')\right]\Big|_{\sigma=-\infty}^{\sigma=+\infty} + \int d\sigma\left(\frac{\tilde{\chi}}{\omega}\theta'\right)\Big|_{\tau=-\infty}^{\tau=+\infty} \right\}
\end{aligned} \tag{7}$$

where the expression for $\hat{R}$ has been used, and $\mu = 0, 1, 2$, with $\eta^{\mu\nu} = (1, -1, -1)$. In the above, dots and primes denote differentiation with respect to time and space respectively. The canonical momenta associated with the fields $\{\omega, \theta, \psi_\mu, f_i\}$ are

$$P_\omega = 0 \tag{8}$$

$$P_\theta = 0 \tag{9}$$

$$P_0 = -\frac{\dot{\psi}_0}{\omega} + \frac{\theta\psi_0'}{\omega} + \frac{\sqrt{\kappa}\theta'}{\omega} \tag{10}$$

$$P_1 = -\frac{\dot{\psi}_1}{\omega} - \frac{\theta\psi_1'}{\omega} - \frac{\sqrt{\kappa}\theta'}{\omega} \tag{11}$$

$$P_2 = \frac{\dot{\psi}_2}{\omega} - \frac{\theta\psi_2'}{\omega} \tag{12}$$

$$\pi_i = \frac{f_i}{2\omega} - \frac{\theta f_i'}{2\omega} \tag{13}$$

Clearly (8) and (9) are primary constraints and $\omega(x)$ and $\theta(x)$ play the role of lagrange multipliers. Then action (7) becomes

$$\begin{aligned} S &= \int d\tau \int d\sigma \left[\dot\psi_0 P_0 + \dot\psi_1 P_1 + \dot\psi_2 P_2 + \sum_{i=1}^{N} \dot f_i \pi_i - (\omega \mathcal{H}_\omega + \theta \mathcal{H}_\theta) \right] \\ &\quad - \left\{ \int d\Sigma \sqrt{\pm h} K \tilde\chi + \int d^2\sigma \partial_\alpha \left[\sqrt{-\hat g} \hat g^{\alpha\beta} \tilde\chi \partial_\beta \rho \right] \right. \\ &\quad + \int d\tau \left[\frac{\tilde\chi}{\omega}(\omega\omega' - \theta\theta') - \sqrt{\kappa}(\psi_0' - \psi_1')\omega \right. \\ &\quad \left. \left. + \sqrt{\kappa}(P_0 + P_1)\theta \right] \Big|_{\sigma=-\infty}^{\sigma=+\infty} + \int d\sigma \left(\frac{\tilde\chi}{\omega} \theta' \right) \Big|_{\tau=-\infty}^{\tau=+\infty} \right\} \end{aligned} \tag{14}$$

where

$$\begin{aligned} \mathcal{H}_\omega &= -\frac{1}{2}(P_0^2 + {\psi_0'}^2) + \frac{1}{2}(P_1^2 + {\psi_1'}^2) + \frac{1}{2}(P_2^2 + {\psi_2'}^2) \\ &\quad + \sqrt{\kappa}(\psi_0'' - \psi_1'') - 2\lambda^2 e^{\frac{2}{\sqrt{\kappa}}(\psi_0 - \psi_2)} + \sum_{i=1}^{N} (\pi_i^2 + \frac{1}{4} f_i'^2) = 0 \end{aligned} \tag{15}$$

$$\mathcal{H}_\theta = P_0 \psi_0' + P_1 \psi_1' + P_2 \psi_2' - \sqrt{\kappa}(P_0' + P_1') + \sum_{i=1}^{N} \pi_i f_i' = 0 \tag{16}$$

are secondary constraints, which satisfy the following Poisson brackets:

$$\begin{aligned} \{\mathcal{H}_\omega(\sigma), \mathcal{H}_\omega(\sigma')\} &= [\mathcal{H}_\omega(\sigma) + \mathcal{H}_\omega(\sigma')] \partial_\sigma \delta(\sigma - \sigma') \\ \{\mathcal{H}_\theta(\sigma), \mathcal{H}_\theta(\sigma')\} &= [\mathcal{H}_\theta(\sigma) + \mathcal{H}_\theta(\sigma')] \partial_\sigma \delta(\sigma - \sigma') \\ \{\mathcal{H}_\omega(\sigma), \mathcal{H}_\theta(\sigma')\} &= [\mathcal{H}_\omega(\sigma) + \mathcal{H}_\omega(\sigma')] \partial_\sigma \delta(\sigma - \sigma') \end{aligned} \tag{17}$$

In the conformal gauge ($\omega = 1, \theta = 0$), $g_{++} = g_{--} = 0$, $g_{+-} = -\frac{1}{2}e^{2\rho}$, the action (14) can be reduced to

$$\begin{aligned} S &= \int d\tau \int d\sigma \left[\dot\psi_0 P_0 + \dot\psi_1 P_1 + \dot\psi_2 P_2 + \sum_{i=1}^{N} \dot f_i \pi_i - \mathcal{H}_\omega \right] \\ &\quad - \int d\tau \left[2\tilde\chi\rho' - \tilde\chi' \right] \Big|_{\sigma=-\infty}^{\sigma=+\infty} \end{aligned} \tag{18}$$

In the derivation of action (18), we have exploited the induced metric and extrinsic curvature on the spacelike boundary Σ defined by $h = e^{2\rho}$ and $K = \nabla_\alpha n^\alpha$, while the induced metric and extrinsic curvature on the timelike boundary B defined by $h = -e^{2\rho}$ and $K = \nabla_\alpha \gamma^\alpha$ [20-22], where the timelike unit vector n^α normal to Σ and spacelike unit vector γ^α normal to B are defined in the conformal gauge as [20-22]

$$n^\alpha = (e^{-\rho}, 0), \qquad \gamma^\alpha = (0, e^{-\rho}) \tag{19}$$

with

$$n{\cdot}n = -1, \qquad \gamma{\cdot}\gamma = +1 \tag{20}$$

From (18), the total Hamiltonian can be written as

$$H_T = \int d\sigma \mathcal{H}_\omega + H_\alpha \tag{21}$$

where

$$H_\alpha = \left(2\tilde{\chi}\rho' - \tilde{\chi}'\right)\Big|_{\sigma=-\infty}^{\sigma=+\infty} \tag{22}$$

Now since $\mathcal{H}_\omega = 0$ (weakly) is a constraint of the theory, the energy is entirely given by the boundary term. Then we have our expression for the ADM mass:

$$E_{ADM} = \Delta\left(\tilde{\chi}\rho' - \frac{1}{2}\tilde{\chi}'\right)\Big|_{-\infty}^{\infty} \tag{23}$$

In the derivation of Eq.(23), we have defined $\Delta\tilde{\chi} = \tilde{\chi} - \tilde{\chi}_{LDV}$, $\Delta\rho = \rho - \rho_{LDV}$, where $\tilde{\chi}_{LDV}$, ρ_{LDV} are the linear dilaton vacuum solution, since we should measure energy relative to the linear dilaton vacuum. And from Eq.(22) to (23), one half factor has been recovered which was omitted previously.

In the asympotically Minkowski coordinates $\sigma^\pm$ which connect with the Kruskal-like coordinates $x^\pm$ by $e^{\pm\lambda\sigma^\pm} = \pm\lambda x^\pm$, we have $\rho = \phi + \lambda\sigma$, so Eq.(23) can be written as

$$E_{ADM} = \Delta\left(\lambda\tilde{\chi} + \tilde{\chi}\phi' - \frac{1}{2}\tilde{\chi}'\right)\Big|_{-\infty}^{\infty} \tag{24}$$

when dropping all quantum corrections, i.e., $\tilde{\chi} = e^{-2\phi}$, then we have

$$E_{ADM} = \Delta\left[e^{-2\phi}(\lambda + 2\phi')\right]\Big|_{-\infty}^{\infty} \tag{25}$$

Eq.(25) shows that at classical level our mass formula can be reduced to that of refs.[5,6].

From (6), Eq.(24) can be written as

$$E_{ADM} = \Delta \left[\lambda\sqrt{\kappa}(\psi_0 - \psi_1) + \sqrt{\kappa}(\psi_0 - \psi_1)\phi' - \frac{\sqrt{\kappa}}{2}(\psi_0' - \psi_1') \right] \Bigg|_{-\infty}^{\infty} \tag{26}$$

In the Kruskal-like coordinates $x^{\pm}$, the hidden dynamical field ψ_1 can be chosen as [10-12]

$$\psi_1(x) = -\frac{\sqrt{\kappa}}{2} \ln(-\lambda^2 x^+ x^-) \tag{27}$$

Since Eq.(26) contains the dilaton field ϕ, as pointed out in refs.[1,23], we have in the weak-coupling region ($e^{2\phi} \ll 1$), i.e., $\sigma \to \infty$, $\phi_2 \simeq \frac{1}{\sqrt{\kappa}} e^{-2\phi}$, while in the strong-coupling region ($e^{2\phi} \gg 1$), i.e., $\sigma \to -\infty$, $\psi_2 \simeq \frac{\sqrt{\kappa}}{2}\phi$, and we will apply this approximation in the following calculations for the ADM and Bondi mass.

Now let us evaluate the ADM mass for the static solution [9,23,24]:

$$\begin{aligned} \sqrt{\kappa}(\psi_0 - \psi_1) &= \frac{m_0}{\lambda} + \sqrt{\kappa}(\psi_0 - \psi_1)_{LDV} \\ \sqrt{\kappa}\psi_2 &= \frac{m_0}{\lambda} + (\sqrt{\kappa}\psi_2)_{LDV} \end{aligned} \tag{28}$$

where m_0 is a constant, and $(\psi_0 - \psi_1)_{LDV}$ and $(\psi_2)_{LDV}$ are the quantum solutions corresponding to the linear dilaton vacuum of the classical theory which are given by

$$\begin{aligned} \sqrt{\kappa}(\psi_0 - \psi_1)_{LDV} &= e^{\lambda(\sigma^+ - \sigma^-)} + \frac{\kappa}{4}\lambda(\sigma^+ - \sigma^-) \\ \sqrt{\kappa}(\psi_2)_{LDV} &= e^{\lambda(\sigma^+ - \sigma^-)} - \frac{\kappa}{4}\lambda(\sigma^+ - \sigma^-) \end{aligned} \tag{29}$$

where we have used Eq.(27) for ψ_1. From (26) and the approximation for ϕ in the different regions, then the ADM mass in the absence of the RST boundary for the static solution is

$$E_{ADM} = 0 \tag{30}$$

which is consistent with the result in ref.[6].

In the case of an incoming shock wave of f matter $T^f_{++} = (ae^{\lambda\sigma_0^+}/\lambda)\delta(\sigma^+ - \sigma_0^+)$, $T^f_{--} = 0$ [1], the conformal frame in which the solution is asymptotically Minkowski is related to the σ frame by $\bar{\sigma}^+ = \sigma^+$, $\bar{\sigma}^- = -(1/\lambda)\ln(e^{-\lambda\sigma^-} - \frac{a}{\lambda})$, and the solutions are [9,22]

$$\sqrt{\kappa}(\psi_0 - \psi_1) = \begin{cases} \frac{m}{\lambda} + e^{\lambda(\bar{\sigma}^+ - \bar{\sigma}^-)} + \frac{\kappa}{4}\ln\left[e^{\lambda\bar{\sigma}^+}\left(e^{-\lambda\bar{\sigma}^-} + \frac{a}{\lambda}\right)\right], & \bar{\sigma}^+ \geq \bar{\sigma}_0^+, \\ e^{\lambda\bar{\sigma}^+}\left(e^{-\lambda\bar{\sigma}^-} + \frac{a}{\lambda}\right) + \frac{\kappa}{4}\ln\left[e^{\lambda\bar{\sigma}^+}\left(e^{-\lambda\bar{\sigma}^-} + \frac{a}{\lambda}\right)\right], & \bar{\sigma}^+ < \bar{\sigma}_0^+ \end{cases} \tag{31}$$

and

$$\sqrt{\kappa}\psi_2 = \begin{cases} \frac{m}{\lambda} + e^{\lambda(\bar{\sigma}^+ - \bar{\sigma}^-)} - \frac{\kappa}{4}\ln\left[e^{\lambda\bar{\sigma}^+}\left(e^{-\lambda\bar{\sigma}^-} + \frac{a}{\lambda}\right)\right], & \bar{\sigma}^+ \geq \bar{\sigma}_0^+, \\ e^{\lambda\bar{\sigma}^+}\left(e^{-\lambda\bar{\sigma}^-} + \frac{a}{\lambda}\right) - \frac{\kappa}{4}\ln\left[e^{\lambda\bar{\sigma}^+}\left(e^{-\lambda\bar{\sigma}^-} + \frac{a}{\lambda}\right)\right], & \bar{\sigma}^+ < \bar{\sigma}_0^+ \end{cases} \tag{32}$$

where we have exploited Eq.(27) for ψ_1, and $m = ae^{\lambda\sigma_0^+}$ is the mass of the classical black hole. In evaluating the ADM mass, we need to compare with the LDV solution in the same conformal frame, that is, (29) with σ replaced by $\bar{\sigma}$. Then the ADM mass without the RST boundary for the dynamical solutions (31) and (32) is given by

$$E_{ADM}(\bar{\tau}) = m - \frac{\kappa\lambda^2}{8}(\bar{\tau} + \bar{\sigma})\Big|_{\bar{\sigma}\to-\infty} - \frac{\kappa\lambda}{8}(\ln\frac{a}{\lambda} + 1) \tag{33}$$

The above equation shows that the ADM mass is infinite at quantum level, whereas it is well-defined at classical level ($\kappa = 0$). The divergent part comes from the fact that due to the quantum anomaly the solution in the region $\bar{\sigma}^+ < \bar{\sigma}_0^+$ does not go to the LDV as $\bar{\sigma} \to -\infty$ once we insist that the solution for $\bar{\sigma}^+ > \bar{\sigma}_0^+$ is asymptotically Minkowski.

As we know, the Bondi mass describes the total energy minus the energy that has been radiated away up to a given retarded time, so it should be evaluated on a line which is asymptotic to $\bar{\sigma}^- = constant$ at $\bar{\sigma}^+ \to \infty$ and to $\bar{\sigma}^+ = \bar{\sigma}_1^+ < \bar{\sigma}_0^+$ on $\bar{\sigma}^- \to \infty$, then in analogy with the expression (26) for the ADM mass, the Bondi mass can be defined as [6]

$$\begin{aligned} E_{Bondi} \;=\; & \Delta\Big[\lambda\sqrt{\kappa}(\psi_0 - \psi_1) + \sqrt{\kappa}(\psi_0 - \psi_1)(\partial_+ - \partial_-)\phi \\ & - \frac{\sqrt{\kappa}}{2}(\partial_+ - \partial_-)(\psi_0 - \psi_1)\Big]\Big|_{\bar{\sigma}^- = -\infty, \bar{\sigma}^+ = \bar{\sigma}_1^+}^{\bar{\sigma}^+ = +\infty} \end{aligned} \tag{34}$$

From (31), (32)and (34), we have

$$E_{Bondi} = m - \frac{\kappa\lambda}{4}\ln\left(1 + \frac{a}{\lambda}e^{\lambda\bar{\sigma}^-}\right) - \frac{\kappa\lambda^2}{8}\bar{\sigma}_1^+ - \frac{\kappa\lambda}{8}(\ln\frac{a}{\lambda} + 1) \tag{35}$$

The above equation shows that due to the existence of the hidden dynamical field ψ_1, the original divergent Bondi mass [6] becomes convergent and unbounded from below, however, when the RST boundary is imposed, the Bondi mass will have a lower bound.

Now we consider the situation with the RST boundary as done in ref.[9]. The RST boundary conditions put $\partial_\pm\psi_2 = f = 0$ on the critical curve, which is regarded as the left boundary of spacetime whenever it is timelike. The RST boundary is at [9]

$$\psi_2 = \psi_2^c = \frac{1}{\sqrt{\kappa}} \left[\frac{\kappa}{4} - \frac{\kappa}{4} \ln \frac{\kappa}{4} \right] \tag{36}$$

As pointed out in ref.[6], there is no solution to the boundary curve equation for the static case (28) with $m_0 > 0$. This means that the boundary trajectory stays behind the classical event horizon ($x^- = 0$) [25]. So the left boundary of the spacetime can be set at the negative infinite space end $\bar{\sigma} \to -\infty$. Then we have

$$E_{ADM} = 0 \tag{37}$$

For the collapse case with the RST boundary, the ADM mass is given by

$$E_{ADM} = m - \frac{\kappa\lambda}{4}(3 + \ln \frac{\kappa}{4}) \cdot \left[1 + \sqrt{1 + \frac{\kappa\lambda^2}{a^2} e^{-2\lambda\bar{\tau}}} \right]^{-1} \tag{38}$$

where we have used the boundary trajectory equation (36) $e^{\lambda\bar{\sigma}_B^+}(e^{-\lambda\bar{\sigma}_B^-} + \frac{a}{\lambda}) = \frac{\kappa}{4}$, i.e., $e^{\lambda\bar{\sigma}_B} = \frac{a}{\lambda} e^{\lambda\bar{\tau}} \left[-1 + \sqrt{1 + \frac{\kappa\lambda^2}{a^2} e^{-2\lambda\bar{\tau}}} \right]$ for the dynamical solution (32), while $e^{\lambda(\bar{\sigma}_B^+ - \bar{\sigma}_B^-)} = \frac{\kappa}{4}$ for the LDV solution (29), and $(\psi_0 - \psi_1)^c = \frac{\kappa}{4} + \frac{\kappa}{4} \ln \frac{\kappa}{4}$, $(\psi_2)^c = \frac{\kappa}{4} - \frac{\kappa}{4} \ln \frac{\kappa}{4}$ at the RST boundary.

Eq.(38) shows that there is a new contribution to the ADM mass from the RST boundary due to the existence of the hidden dynamical field ψ_1. When $\bar{\tau} \to -\infty$, the quantum ADM mass goes to the mass of the collapsing matter.

The corresponding Bondi mass with RST boundary conditions can be evaluated from (34). The lower limit in (34) is replaced by a point on the critical curve $(\psi_0 - \psi_1) = (\psi_0 - \psi_1)^c$, $\psi_2 = (\psi_2)^c$ for $\bar{\sigma}^+ < \bar{\sigma}_0^+$. At the upper end, however, there are two regions to consider. Calling the point where the apparent horizon and the critical curve intersect $(\bar{\sigma}_s^+, \bar{\sigma}_s^-)$, we have the region $\bar{\sigma}^- < \bar{\sigma}_s^-$ (region I of RST; see ref.[9] for figure) and the region between the timelike boundary and $\bar{\sigma}^- = \bar{\sigma}_s^-$ (region II of RST). In region II the black hole has decayed and the solution is taken to be the LDV. In region I the solution is the collapsing solution for $\bar{\sigma}^+ > \bar{\sigma}_0^+$. Thus we have [9-12]

$$\sqrt{\kappa}(\psi_0 - \psi_1) = \begin{cases} \frac{m}{\lambda} + e^{\lambda(\bar{\sigma}^+ - \bar{\sigma}^-)} + \frac{\kappa}{4} \ln \left[e^{\lambda\bar{\sigma}^+} \left(e^{-\lambda\bar{\sigma}^-} + \frac{a}{\lambda} \right) \right], & \bar{\sigma}^- < \bar{\sigma}_s^-, \\ e^{\lambda(\bar{\sigma}^+ - \bar{\sigma}^-)} - \frac{\kappa}{4} \ln \left(e^{\lambda\bar{\sigma}^+} \cdot e^{-\lambda\bar{\sigma}^-} \right) + \frac{\kappa}{2} \ln \left[e^{\lambda\bar{\sigma}^+} \left(e^{-\lambda\bar{\sigma}^-} + \frac{a}{\lambda} \right) \right], & \bar{\sigma}^- \geq \bar{\sigma}_s^- \end{cases} \tag{39}$$

and

$$\sqrt{\kappa}\psi_2 = \begin{cases} \frac{m}{\lambda} + e^{\lambda(\bar\sigma^+ - \bar\sigma^-)} - \frac{\kappa}{4}\ln\left[e^{\lambda\bar\sigma^+}\left(e^{-\lambda\bar\sigma^-} + \frac{a}{\lambda}\right)\right], & \bar\sigma^- < \bar\sigma_s^-, \\ e^{\lambda(\bar\sigma^+ - \bar\sigma^-)} - \frac{\kappa}{4}\ln\left(e^{\lambda\bar\sigma^+}\cdot e^{-\lambda\bar\sigma^-}\right), & \bar\sigma^- \geq \bar\sigma_s^- \end{cases} \tag{40}$$

From (34),(39) and (40), we have

$$E_{Bondi} = \begin{cases} m - \frac{\kappa\lambda}{4}\ln\left(1 + \frac{a}{\lambda}e^{\lambda\bar\sigma^-}\right) - \frac{\kappa\lambda}{8}(3 + \ln\frac{\kappa}{4})\left[1 + \frac{\lambda}{a}e^{-\lambda\bar\sigma^-}\right]^{-1}, & in\ \ I \\ \frac{\kappa\lambda}{4}\left(1 + \frac{\lambda}{a}e^{-\lambda\bar\sigma^-}\right)^{-1} - \frac{\kappa\lambda}{8}(3 + \ln\frac{\kappa}{4})\left(1 + \frac{\lambda}{a}e^{-\lambda\bar\sigma^-}\right)^{-1}, & in\ \ II \end{cases} \tag{41}$$

Like the ADM mass, there is also a new contribution to the Bondi mass from the boundary due to the hidden dynamical field ψ_1. It can be seen from (41) that in the presence of the RST boundary, the Bondi mass has a lower bound. For $\bar\sigma^- \to -\infty$, $E_{Bondi} \to m$; while for $\bar\sigma^- \to +\infty$, $E_{Bondi} \to -\frac{\kappa\lambda}{8}(1 + \ln\frac{\kappa}{4})$, that is, E_{Bondi} turns to be negative. This is not surprising, since even at $\bar\sigma^- = \bar\sigma_s^-$, E_{Bondi} has already turned to be negative. On the other hand, the energy flow is discontinuous at $\bar\sigma^- = \bar\sigma_s^- = \lambda^{-1}\ln[\frac{\lambda}{a}(e^{\frac{4m}{\kappa\lambda}} - 1)]$:

$$E_{Bondi}(\bar\sigma_s - 0) - E_{Bondi}(\bar\sigma_s + 0) = -\frac{\kappa\lambda}{4}\left(1 - e^{-\frac{4m}{\kappa\lambda}}\right) \tag{42}$$

This is just the effect of the thunderpop of RST and is caused by the fact that when the collpase solution is matched to the LDV along the null line $\bar\sigma^- = \bar\sigma_s^-$, the result is continuous but not smooth. Thus Eq.(42) shows that our new mass formula derived from Hawking and Horowitz's conjecture is quite reasonable.

In summary, the ADM and Bondi mass for the RST model have been first discussed from Hawking and Horowitz's argument. The boundary terms in the Hamiltonian come directly from the boundary terms in the action, and do not need to be added "by hand". The result shows that the previous expression for the ADM and Bondi mass actually needs to be modified at the quantum level. But our new mass formula can be reduced to that of refs.[5,6] at the classical level. It has been shown that there is a new contribution to the ADM and Bondi mass from the hidden dynamical field ψ_1. In the absence of the RST boundary, the ADM mass for the static solution is zero, while for the dynamical case with collapsing matter, the ADM mass is infinite and positive, whereas the Bondi mass is finite. On the other hand, in the presence of the RST boundary, it has been found that the ADM mass for the static solutions with $m_0 > 0$ is zero, while for the dynamical case, there is a new contribution to the ADM mass from the RST boundary, which is just the consequence that the hidden dynamical field modifies the total energy. However, the Bondi mass begins to turn into negative at an intermediate time, and is discontinuous across a certain null line $\bar{\sigma}^- = \bar{\sigma}_s^-$, that is, the thunderpop of the RST model can also be reflected from our new Bondi mass formula.

Acknowledgement

This work was supported in part by the European Union Under the Human Capital and Mobility programme. J.-G. Z. thanks the Alexander von Humboldt Foundation for financial support in the form of a research fellowship. Y.-G. M. was supported by the Natural Science Foundation of Fujian Province under the grant No.A95009.

References

[1] C.Callan, S.Giddings, J.Harvey and A.Strominger, *Phys.Rev.*__D45__(1992)R1005.

[2] Y.Park and A.Strominger, *Phys.Rev.*__D47__(1993)1569.

[3] E.Witten, *Phys.Rev.*__D44__(1991)314.

[4] S.Giddings and A.Strominger, *Phys.Rev.*__D47__(1993)2454.

[5] A.Bilal and I.Kogan, *Phys.Rev.*__D47__(1993)5408.

[6] S. de Alwis, *Phys.Rev.*__D49__(1994)941.

[7] D.Bak, D.Cangemi and R.Jackiw, *Phys.Rev.*__D49__(1994)5173.

[8] T.Regge and C.Teitelboim, *Ann.Phys.(N.Y.)*__88__(1974)286.

[9] J.G.Russo, L.Susskind and L.Thorlacious, *Phys.Rev.*__D46__(1992)3444; *Phys.Rev.*__D47__(1993)533.

[10] J.-G.Zhou, F.Zimmerschied, J.-Q.Liang and H.J.W.Müller-Kirsten, *A new look at the RST model*, KL-TH-95/13.

[11] J.D.Hayward, *Entropy in the RST model*, gr-qc/9412065.

[12] R.C.Myers, *Phys.Rev.*__D50__(1994)6412.

[13] S.W.Hawking and G.T.Horowitz, *The gravitational Hamiltonian, action, entropy and surface terms*, gr-qc/9501014.

[14] S.W.Hawking and S.F.Ross, *Duality between electric and magnetic black holes*, hep-th/9504019.

[15] C.G.Torre, *Phys.Rev.*__D40__(1989)2588.

[16] T.Fujiwara, Y.Igarashi and J.Kubo, *Phys.Lett.*__B316__(1993)66.

[17] D.Louis-Martinez, J.Gegenberg and G.Kunstatter, *Phys.Lett.*__B321__(1994)193.

[18] J.-G.Zhou, H.J.W.Müller-Kirsten and M.-Z.Yang, *Phys.Rev.*__D51__(1995)R314.

[19] W.T.Kim, J.Lee and Y.-J.Park, *Phys.Lett.*__B347__(1995)217.

[20] J.D.Brown and J.W.York, Jr., *Phys.Rev.*__D47__(1993)1407.

[21] E.A.Martinez, *Phys.Rev.*__D51__(1995)5732.

[22] C.Misner, K.Thorne and J.Wheeler, *Gravitation* (Freeman, San Francisco, 1973).

[23] S. de Alwis, *Phys.Lett.*__B289__(1992)278; *Phys.Lett.*__B300__(1993)330; *Phys.Rev.*__D46__(1992)5429.

[24] A.Bilal and C.Callan, *Nucl.Phys.*__B394__(1993)73.

[25] K.Schoutens, H.Verlinde and E.Verlinde, *Phys.Rev.*__D48__(1993)2670.