

FINAL REPORT

1 General Information

DFG reference number: DO 2214/3-1

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Project title: Efficient analysis of shell structures defined in highly complex CAD models (Effiziente Berechnung von Schalenstrukturen aus hochkomplexen CAD-Modellen)

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Name(s) of the co-applicants: - none -

Name(s) of the cooperation partners: - none -

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2 Summary

English: The main objective of this project is developing an efficient shell element for the large deformation analysis of complex thin-walled structures based on the geometry description of Computer-Aided design (CAD) software. The intended field of application are CAD models of built structures, but applications in other fields of engineering are akin possible. The basic idea is to combine exact geometry description by Non-Uniform Rational B-splines (NURBS) with the interesting properties of the spectral element method (SEM), where nodal points, i.e. discrete points where all unknown quantities are defined, and integration points, i.e. points where the relevant equations are evaluated, coincide. The location of these points and the nodal normal vectors are computed exactly from the CAD model. The discretization effort is kept minimal since the patch-wise definition of the CAD domains is retained. Interpolation of unknown quantities and derivatives thereof are obtained using Lagrange basis functions, which provide a proper and stable base also for high order computations, while the use of NURBS basis functions for the description of the shape of the elements removes the geometrical approximation error.

The implemented SEM shell formulation overcomes the major shortcomings of pure isogeometric (IGA) shell formulations, where both geometry and unknowns are interpolated by NURBS functions. The numerical examples show that the developed SEM shell yields stable condition numbers and significantly alleviates locking effects for high orders of basis functions. As expected, a simple rotational formulation is sufficient in SEM to obtain results with comparable accuracy as an IGA shell with complicated rotational formulation. In contrast to the initial hypothesis, we discovered that the intended non-isoparametric formulation using both Lagrange and NURBS basis functions is not required to obtain high accuracy results. The specially developed benchmark parcour has shown that the geometrical approximation error is much smaller than the interpolation error. Thus, we adapted the work programme of the project and implemented an isoparametric spectral shell element (SEMI) formulation. The results of the benchmark parcour revealed that for all examples, the SEMI shell performed as least as good as the IGA shell. Surprisingly, we discovered that especially for complicated geometries, i.e. high and highly changing curvature, the SEMI formulation performed more robust and accurate as the IGA shell. By exploiting the Kronecker-Delta property of SEM, we could show that significant gains in computational costs of element stiffness matrices are possible in comparison to IGA. The project provided a deep understanding of the significance of exact geometry representation, which are of high interest to the shell formulation community. In combination with the shown efficiency of SEM, this paves the way for high efficiency computations of complex geometries using SEM formulations.

Deutsch: Das Ziel dieses Projekts ist die Entwicklung effizienter Schalen-Elemente für die Berechnung komplexer dünnwandiger Strukturen, basierend auf der Geometriebeschreibung von Computer-Aided Design (CAD) Software. Das primäre Anwendungsgebiet sind CAD-Modelle von Bauwerken, jedoch sind auch Anwendungen in anderen Ingenieurdisziplinen möglich. Die Grundidee ist, die exakte Geometriebeschreibung durch Non-Uniform Rational B-Splines (NURBS) mit den interessanten Eigenschaften der Spektralen-Elemente-Methode (SEM) zu kombinieren, bei der Knoten, d. h. diskrete Punkte, an denen die unbekannt GröÙen definiert sind, und Integrationspunkte, d. h. Punkte, an denen die relevanten Gleichungen ausgewertet werden, zusammenfallen. Die Lage dieser Punkte und die Knoten-Normalenvektoren werden exakt aus dem CAD-Modell berechnet. Der Diskretisierungsaufwand wird minimiert, da die CAD-Teilgebiete beibehalten werden. Die Interpolation unbekannter GröÙen und deren Ableitungen erfolgt mit Lagrange-Basisfunktionen, die eine verlässliche Grundlage auch für Berechnungen mit hohen Ansatzordnungen bilden, während die Beschreibung der Elementform durch NURBS-Basisfunktionen den geometrischen Approximationsfehler beseitigt.

Die implementierte SEM-Schalenformulierung überwindet einige Nachteile isogeometrischer (IGA) Schalenelemente, bei denen Geometrie und Unbekannte durch NURBS-Funktionen interpoliert werden. Die numerischen Beispiele zeigen, dass die SEM-Schale stabile Konditionszahlen liefert und Locking-Effekte bei hohen Ordnungen der Basisfunktionen signifikant verringert. Wie erwartet reicht eine einfache Rotationsformulierung für SEM aus, um Ergebnisse vergleichbarer Genauigkeit wie eine IGA-Schale mit komplizierter Rotationsformulierung zu erzielen. Im Gegensatz zur anfänglichen Hypothese ist die nicht-isoparametrische Formulierung, die sowohl Lagrange- als auch NURBS-Basisfunktionen verwendet, nicht erforderlich, um präzise Ergebnisse zu erzielen. Der entwickelte Benchmark-Parcours hat gezeigt, dass der geometrische Approximationsfehler viel kleiner ist als der Interpolationsfehler. Daher haben wir das Arbeitsprogramm des Projekts angepasst und eine isoparametrische Spektral-Schalenformulierung (SEMI) implementiert. Im Benchmark-Parcours schneidet die SEMI-Schale in allen Beispielen mindestens genauso gut wie die IGA-Schale ab. Überraschenderweise ist die SEMI-Schale insbesondere bei Geometrien mit hoher und stark variierender Krümmung robuster und genauer als die IGA-Schale. Durch die Ausnutzung der Kronecker-Delta-Eigenschaft der SEM sind signifikante Einsparungen bei den Berechnungskosten der Elementsteifigkeitsmatrizen im Vergleich zu IGA möglich. Das Projekt hat ein tiefes Verständnis für die Bedeutung der exakten Geometrie-Darstellung vermittelt, was für die Schalen-Community von großem Interesse ist. In Kombination mit der gezeigten Effizienz der SEM ebnet dies den Weg für hoch-effiziente Berechnungen komplexer Geometrien unter Verwendung von SEM-Formulierungen.

3 Progress Report

Background and objectives: The main objective of the proposal was developing a fast and accurate shell element for the analysis of complex shell structures. To achieve this goal, a super spectral [1] non-isoparametric shell element in conjunction with a NURBS-based definition of the geometry was developed. The shape of the elements is directly defined by NURBS surfaces, which makes the procedure free from inexact definition of geometry. The Lagrange basis functions of the spectral elements provide a proper base for a straightforward and accurate interpolation of rotations since the nodes of spectral elements and integration Gauss-Lobatto-Legendre (GLL) points coincide [2]. The objective was to overcome the main shortcomings of isogeometric analysis (IGA) [3], which are

- (i) the lack of a proper rotational formulation for shell elements
- (ii) efficient and robust methods against locking
- (iii) proper treatment of trimmed elements
- (iv) efficient and accurate coupling of patches,

for the field of Reissner-Mindlin shell formulations. The proposal was split into five work packages. The first work package was to develop and implement a purely displacement-based shell formulation. By using SEM with GLL integration, we anticipated that the usage of the simple discrete rotation concept, see e. g. [4], would yield highly accurate results and solve shortcoming (i) since artificial thinning is prevented by the Kronecker-Delta property. The usage of high order computations was intended to alleviate locking effects (ii). Due to the slower deterioration of the condition number in comparison to IGA, high-order computations are possible in SEM, see [P1] and the preliminary studies in the proposal. In the second work package a reparameterization of trimmed NURBS patches to untrimmed SEM triangles and quadrilaterals was proposed to handle meshes with trimmed patches (iii). In work package 3, we aimed at coupling the SEM elements of non-conforming NURBS patches with a dual mortar method using NURBS boundary curves as intermediary. In work package 4 an adapted Newton method should improve the nonlinear iteration convergence speed. Verification with complex multi-patch CAD models and documentation formed work package 5.

Project-specific results and findings: In the beginning of work package 1 we developed a geometrically linear spectral shell element based on NURBS geometry definition (SEMN). The theoretical development and implementation in Python worked as expected. Studying several benchmark examples, we could elaborate that SEMN yields highly precise results for p-refinement and exhibits a significantly slower growing condition number than IGA. Exemplarily, we provide the most important results to describe the project development. Further results can be found in detail in [P1]. We compare our results to computations using the iso-

geometric shell formulation proposed in [5], abbreviated as IGA-(RMC) if the more complicated continuous rotation interpolation is used, and as IGA-RMD if the same rotational concept as in SEMN is used. The accuracy of the computations of the Scordelis-Lo roof, a well-known benchmark example, is compared in Fig. 1b. SEMN is as accurate as IGA, beginning from order $p = 4$, and works stable also for high orders. The failure of IGA for high orders can clearly be attributed to the bad condition numbers, which are depicted in Fig. 2. These results clearly confirmed our hypothesis that SEMN would allow high order computations with high precision.

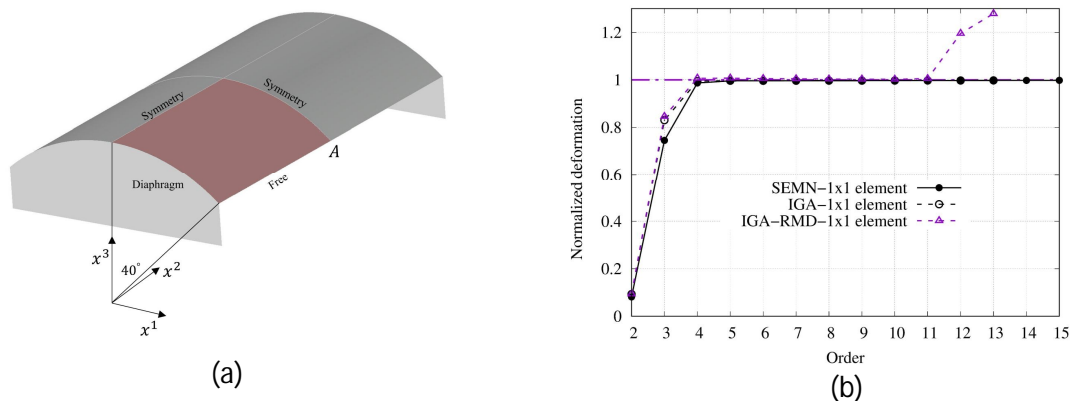


Fig.1. Scordelis-Lo roof: (a) system sketch and (b) normalized deformation for p -refinement vs order of element

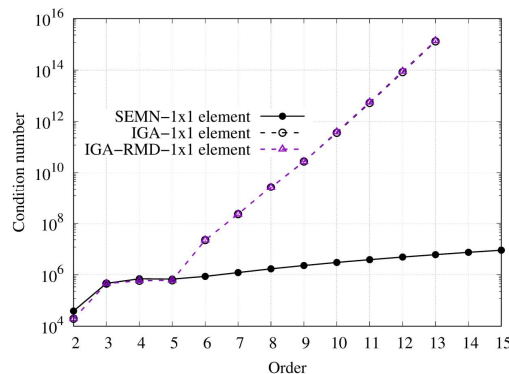


Fig. 2. Scordelis-Lo roof: Condition number of stiffness matrix in p -refinement

A comparison of the circumferential shear force plotted over the x^1 -coordinate at $x^2 = 12.5$ between SEMN and the two IGA formulations is provided in Fig.a and b. The results show that for order $p = 5$, IGA is slightly more precise than SEMN, but for higher orders, the results correspond very well. Despite the very bad condition number of IGA, no oscillations occur for IGA-RMC in Fig. 3a. In contrast to that, IGA-RMD shows slight oscillations for $p = 10$, and severe oscillations for $p = 13$. Reminding that IGA-RMD and SEMN use the same rotational formulation, this clearly shows that the chosen simple discrete rotation formulation of SEMN is sufficient to obtain results that are as precise as the significantly more complex IGA-RMC formulation. This point is a crucial aspect and shows the capabilities of SEMN, as the cost of

continuous rotation modelling constitutes a significant portion of the total analysis cost when compared to a simple discrete formulation (See Tab. 4 in [5]). The results of all other standard benchmark examples are similar and can be found in [P1]. All results have shown that SEMN, where geometry, Jacobian and curvature are exact, is as accurate as IGA for high orders.

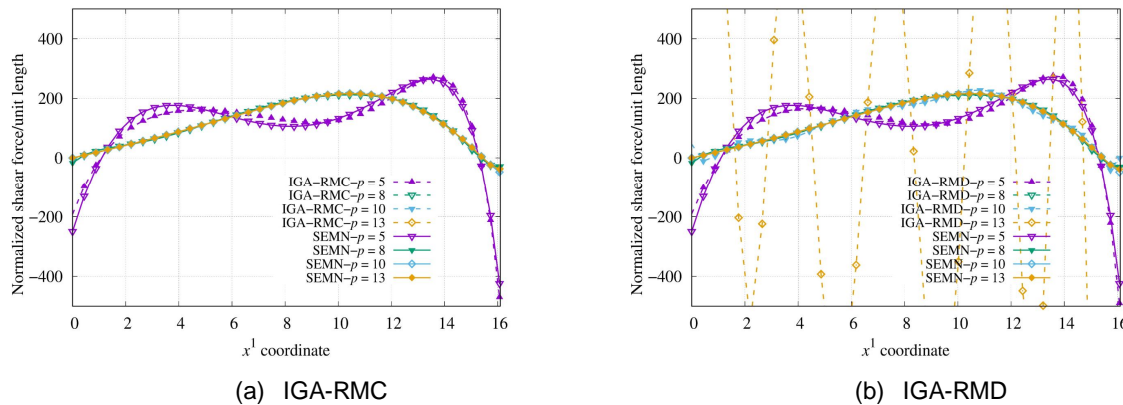


Fig. 3. Scordelis-Lo roof: Circumferential shear force per unit length for IGA and SEMN vs the x^1 coordinate

Quality enhancing measures: As a measure for quality enhancing, we decided to adapt the work programme and implement also a conventional isoparametric spectral shell element (SEMI), where everything is interpolated using Lagrange basis functions. Nevertheless, by computing nodal positions from the NURBS geometry, those are exact, but director vectors and curvature are not. This allows judging about the influence of the exact geometry representation. In addition to addressing well-known benchmarks, where differences between IGA, SEMN and SEMI were quite small, we have strived to solve numerous other problems to investigate the robustness and accuracy of the proposed methods. As a part of this attempt, we considered how the method would perform in analyzing a shell structure with an abrupt curvature variation, modeled by a single patch as shown in Fig. 4a. This scenario is somewhat unusual, and it contrasts with the typical approach of modeling such complex structures with multiple patches and elements. Surprisingly, for this example the best convergence behavior was obtained using the SEMI formulation, see Fig. 4b. The worst result was obtained by IGA, which considers the exact geometry. This result, which strongly contradicted the initial hypothesis of our project, had to be examined thoroughly. Thus, we added several variants for the SEMN shell, for example the use of Gauss integration instead of GLL integration, and the use of the continuous rotation interpolation in SEMN. However, the situation did not change significantly, also considering different benchmarks. The final verdict for the linear shell formulation was that the SEMI shell yields a similar or even better accuracy than an IGA shell formulation. Furthermore, since SEMI performed very similarly to SEMN, the higher effort and complexity of SEMN does not pay off in the considered linear examples.

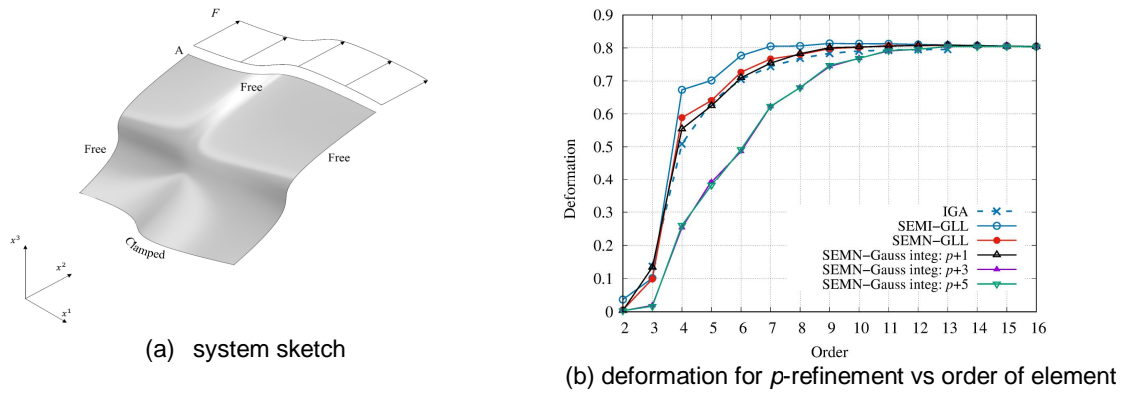


Fig. 4. Newly proposed single patch free form surface with strong change of curvature

Following a discussion on a conference we focussed on the theoretical presence of spurious strains in large rotation problems for non-isoparametric element formulations [6]. The Green-Lagrange strain tensor $\mathbf{E} = \frac{1}{2}(\mathbf{F}^T\mathbf{F} - \mathbf{G})$, with \mathbf{F} and \mathbf{G} being deformation gradient and metric tensors (here in Cartesian system, unit tensor), should vanish for pure rigid body rotations, i.e. $\mathbf{F}^T\mathbf{F} = \mathbf{G}$. In rigid body rotation, the displacement u_i can be calculated using kinematic relations based on undeformed geometry and rotation. The deformation gradient is computed by $F_{ij} = \frac{\partial x_i}{\partial X_j}$, where x_i and X_i are the coordinates of a material point in the deformed and undeformed geometry, respectively. Since in SEMN, $x_i = X_i + u_i$ will be interpolated using both NURBS basis functions (for X_i) and SEM basis functions (for u_i), the accuracy of differentiating the coordinates of material points in the deformed geometry is closely tied to how precisely the undeformed geometry can be estimated by SEM shape functions. If the SEM shape functions could interpolate the geometry exactly, no spurious strains would occur. We devised two types of shape to theoretically test the SEMN formulation on spurious strains for an in-plane rotation of 30 degrees. For the B-spline shape in Fig. 5a, the amount of spurious strains for SEMN was in the range of numerical precision, although being slightly higher than for SEMI. Thus, spurious strains will be no problem for B-spline surfaces, since the function



Fig. 5. (a) B-Spline base geometry (b) NURBS based geometry

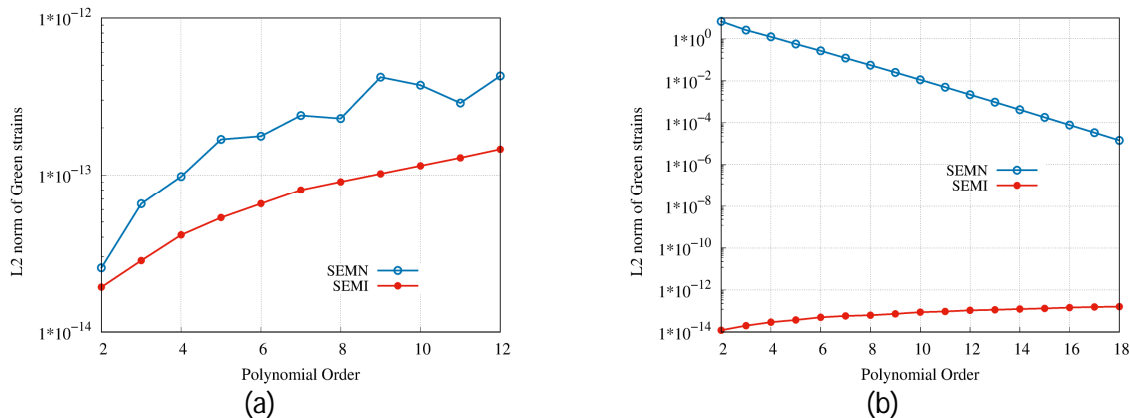


Fig. 6. L_2 norm of Green-Lagrange strains for (a) B-Spline surface (b) NURBS surface

space of B-splines and SEM functions is the same. However, by doing the test for the NURBS shape given in Fig. 5b, which differs only by the weight factor of the two marked points (3 instead of 1), we get a linearly decreasing norm of the spurious strains with considerable size for SEMN, and a numerically zero norm for SEMI, see Fig. 6b.

Deviations from the original concept: Together with the almost equal results of SEMN and SEMI for the linear case, this clearly devised us to deviate from the original concept and to switch over to the SEMI formulation for the formulation of the geometrically nonlinear shell formulation. Thus, in the remaining of the project the SEMI shell was developed. In the formulation of the geometrically nonlinear shell formulation, we developed the so-called “cross scheme” which allows a significant speedup of high-order SEM computations [P2, Sec. 3]. The numerical results of the geometrically nonlinear SEMI shell are very promising. In the standard benchmark examples the accuracy and efficiency of SEMI is comparable to IGA computations with the more complicated rotational formulation [P2, Sec. 4.1 to 4.3]. To shed further light on the advantages of exact geometry representation, we developed several new benchmarks with high and highly changing curvature. We reported the results of the most severe one in [P2, Sec. 4.4]. The results show that the developed SEMI shell formulation yields highly precise results for all kinds of geometries. Furthermore, we have clearly shown that the exact representation of geometry, which is the prominent feature of IGA and our SEMN shell, is not of high importance, since in the proximity of the solution the interpolation approximation error is larger than the geometry approximation error. Our detailed research on the shell formulations took significantly longer than expected due to the unforeseeable difficulties. Thus, we could only finish work package 1, the development of a spectral shell finite element, and partly work package 5, were validation with geometries with complicated curvature changes and documentation of the results were accomplished. However, the re-

sults obtained are in our opinion very promising and will lead to a highly efficient shell formulation in future research. The outcome is two published journal articles in high-class international journals. The published results are of high interest for both the shell formulation and the isogeometric analysis community, since in [P2] we could show interesting details about the necessity of exact geometry representation and about advantages and drawbacks of isogeometric shell analysis.

Handling of research data and software: Two types of data are produced in the project. The developed shell formulation has been implemented in a newly developed Python framework. The source code presents the most important type of data in the project. It is stored and managed using GIT, and freely available on the Gitlab server of RPTU (https://gitlab.rhrk.uni-kl.de/shell_iga_sem). Thus, data security and long-term availability is ensured by central services of RPTU. The theoretical formulations which form the basis for the implementation have been published open-access in renowned international journals. The second data type relevant in the project are input files and simulation results in text form. They are stored on a network storage, hosted and secured by Regional University Computing Center Kaiserslautern-Landau. They will be made available upon request.

Reusability of research data and software: The source code created in the project is freely available on the Gitlab Server of RPTU, see link above, and can be used for further research projects of our and other groups. The newly established benchmark examples for curved shell structures will be a part of our future benchmark parcours, the geometry definition files are also available in Gitlab.

Presentations at conferences: The findings of the project have been presented at five international and one national conference:

- N. Azizi and W. Dornisch. Effect of different rotation and element formulation on the large deformation analysis of shells. 43rd Solid Mechanics Conference (SolMech 2024), Sept. 2024, Wroclaw, Poland.
- W. Dornisch and N. Azizi. On the importance of exact geometry representation for shell geometries with highly changing curvature. 94th GAMM Annual Meeting (GAMM 2024), Mar. 2024, Magdeburg, Germany.
- W. Dornisch and N. Azizi. Vergleich zwischen isogeometrischen und spektralen Reissner-Mindlin Schalelementen. Fachtagung Baustatik– Baupraxis 15, Mar. 2024, Hamburg, Germany.
- N. Azizi and W. Dornisch. Comparison of accuracy between IGA shell and NURBS-based SEM shell. 11th International Conference on IsoGeometric Analysis 2023 (IGA 2023), Jun. 2023, Lyon, France.
- N. Azizi and W. Dornisch. An effort to utilize high order exact geometrically defined Reissner-Mindlin spectral shell elements: Advantages and problems. 9th International Conference on High Order Finite Element and Isogeometric Methods (HOFEIM 2023), Jun. 2023, Larnaca, Cyprus.
- N. Azizi and W. Dornisch. An efficient exact geometrically defined Reissner-Mindlin shell element for analysis of shell structures. 93rd GAMM Annual Meeting (GAMM 2023), Jun. 2023, Dresden, Germany.

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- [3] T.J.R. Hughes, J.A. Cottrell, Y. Bazilevs, Isogeometric analysis: CAD, finite elements, NURBS, exact geometry and mesh refinement, *Comput. Methods Appl. Mech. Eng.* 194 (2005) 4135–4195.
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- [6] J.H. Argyris, D.W. Scharpf, The SHEBA Family of Shell Elements for the Matrix Displacement Method, *Aeronaut. J.* 72 (1968) 873–883.

4 Published Project Results

4.1 Publications with scientific quality assurance

- [P1] N. Azizi and W. Dornisch. A spectral finite element Reissner-Mindlin shell formulation with NURBS-based geometry definition. *Computational Mechanics* 74, 537- 559, 2024. doi: [10.1007/s00466-024-02444-w](https://doi.org/10.1007/s00466-024-02444-w)
- [P2] N. Azizi and W. Dornisch: A rotation-based geometrically nonlinear spectral Reissner-Mindlin shell element, *Finite Elements in Analysis and Design* 251, 104416, 2025, doi: [10.1016/j.finel.2025.104416](https://doi.org/10.1016/j.finel.2025.104416)

4.2 Other publications and published results

- [P3] W. Dornisch and N. Azizi. Vergleich zwischen isogeometrischen und spektralen Reissner-Mindlin Schalenelementen. In B. Oesterle, A. Bögle und W. Weber (eds.): *Berichte der Fachtagung Baustatik-Baupraxis* 15, 333-340, 2024. doi: [10.15480/882.9247](https://doi.org/10.15480/882.9247)

4.3 Patents (applied for and granted)

- none –