Collisions of Spin Wave Envelope Solitons and Self-Focused Spin Wave Packets in Magnetic Films

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Head-on collisions between two-dimensional self-focused spin wave packets and between quasi-one-dimensional spin wave envelope solitons have been directly observed for the first time in yttrium-iron garnet (YIG) films by means of a space- and time-resolved Brillouin light scattering technique. We show that quasi-one-dimensional envelope solitons formed in narrow film strips ("waveguides") retain their shapes after collision, while the two-dimensional self-focused spin wave packets formed in wide YIG films are destroyed in collision. [1]

Since the first observation of a water wave soliton by J.S. Russel [1], solitons in different physical systems were experimentally studied mainly in a quasi-one-dimensional geometry, while propagating along natural (water waves in narrow channels, optical pulses in fibers), or artificial (electromagnetic pulses in artificial nonlinear transmission lines) waveguides (see e.g. [2]). In such waveguides the transverse distributions of fields, associated with soliton propagation, were not of critical importance for the soliton properties. It was shown both numerically [3] and analytically [4] that one of the most fundamental properties of one-dimensional solitons as stable eigen-excitations of nonlinear fields (in contrast with simple solitary waves) is the retention of their shapes after collisions with other solitons. Perturbation theory for one-dimensional solitons [5] has shown that weak dissipation does not qualitatively change the soliton collision properties.

From the theoretical point of view, the qualitative change in the properties of a propagating wave packet takes place when the dimensionality of space is increased. In particular, it was shown in [6] that a wave packet propagating in a two-dimensional space described by a (2+1)-dimensional nonlinear Schrödinger (NLS) equation

$$i\left(\frac{\partial U}{\partial t}+V_g\frac{\partial U}{\partial z}\right)+\frac{1}{2}D\frac{\partial^2 U}{\partial z^2}+S\frac{\partial^2 U}{\partial y^2}-N|U|^2U=-i\omega_r U\ , \qquad (1)$$

is always unstable irrespectively of the relative signs of the dispersion (D) and diffraction (S) coefficients (here U is the dimensionless amplitude of the wave packet envelope, t is time, z and y are the direction of the wave propagation and the transverse in-plane direction, respectively, $V_g = \partial\omega/\partial k_z|_{k_{0z}}$ is the group velocity, $D = \partial^2\omega/\partial k_z^2|_{k_{0z}}$ and $S = \partial\omega/\partial(k_y^2)|_{k_{0z}}$ are the dispersion and diffraction coefficients respectively, $N = \partial\omega/\partial|U|^2|_{k_{0z}}$ is the nonlinear coefficient, ω_r is the phenomenological dissipation parameter, $k_0 = k_{0z}$ is the carrier wavenumber, and $\omega(k_y, k_z, |U|^2)$ is the nonlinear wave dispersion law). If the Lighthill criterion [7] is fulfilled in both in-plane directions ($SN < 0$ and $DN < 0$), allowing two-dimensional self-focusing of the propagating wave packet, the instability is really dramatic, and may lead to a wave collapse [6,8]. It was, however, shown, that by including mechanisms of saturable nonlinearity [9] or dissipation [10] highly localized, self-focused two-dimensional nonlinear wave packets ("bullets") can exist.

It is of fundamental importance to investigate the interaction of these excitations in collision experiments. The experimental investigation of head-on collisions process of *two-dimensional* spin wave bullets and its comparison with collisions of *quasi-one-dimensional* spin wave envelope solitons is the subject of this Letter.

The physical system, where both envelope solitons [11], and wave bullets [10] can be experimentally observed and visualized is a system of long-wavelength dipolar spin waves (SW) propagating along the direction of the bias magnetic field in a tangentially magnetized magnetic garnet film (so-called backward volume magnetostatic waves (BVMSW)). For these waves the magnetic film is a focusing medium along both in-plane directions ($SN < 0$ and $DN < 0$) [11,12].

Although even the best samples of magnetic garnet films, such as monocrystalline yttrium-iron garnet (YIG) films, are much more dissipative than water or optical fibers, it is possible to observe formation, propagation, and collisions of quasi-one-dimensional spin wave envelope solitons in narrow YIG-film strips (SW "waveguides") by means of traditional microwave spectroscopy [11]. In these experiments the microstrip antennae, ori-

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ented perpendicular to the direction of the wave propagation, are used to excite and detect propagating SW pulses. Since the detector antenna integrates the received microwave signals along the antenna length, all the information about the transverse structure of a propagating SW pulse is lost, and only the time profiles of detected pulses can be recorded. In other words, the process of SW envelope soliton propagation in microwave experiments [11] was studied as a purely one-dimensional one with only one geometrical coordinate accessible.

The recent advancement in space- and time-resolved Brillouin light scattering (BLS) technique [11,13] allowed us for the first time to visualize and study the real *two-dimensional* picture of the SW pulse propagation in both SW waveguides and wide samples of YIG films. In particular, using this technique, the formation and propagation of spin wave bullets, i.e., two-dimensional self-focused nonlinear SW packets, collapse of which has been stopped by dissipation, has been recently observed in wide YIG film samples [10].

We report below our experimental studies of the collision scenarios for two types of self-arranged nonlinear SW packets - *quasi-one-dimensional envelope solitons* formed in SW waveguides, and *two-dimensional self-focused wave packets (SW bullets)* formed in wide YIG film samples. Our experiments clearly show that quasi-one-dimensional SW envelope solitons, formed as a result of a dynamic equilibrium between dispersion and nonlinearity, are robust, and, therefore, are not seriously affected by collisions with other solitons. On the contrary, quasi-stable two-dimensional SW bullets, resulting from the combined effects of dispersion, diffraction, nonlinearity and dissipation, are much more fragile, and are almost completely destroyed in head-on collisions.

In our "soliton" experiments we used narrow ($1.5 \times 15 mm^2$) YIG film stripes of the thickness $d = 5.9 \mu m$ (SW waveguides), while in the "bullet" experiments we used wide ($18 \times 26 mm^2$) YIG film samples of the thickness $d = 7.0 \mu m$. All samples were made of high-quality monocrystalline YIG films with unpinned surface spins (ferromagnetic resonance linewidth $2\Delta H = 0.45 Oe$ which corresponds to the relaxation frequency $\omega_r / 2\pi = 2.8 MHz$). YIG films were grown on gadolinium-gallium garnet (GGG) substrates of the orientation (111). The samples were mounted, YIG surface down, on two microstrip antennae of the length $2.5 mm$, and the width $50 \mu m$ separated by a distance of $8 mm$. The YIG film samples were tangentially magnetized in the direction perpendicular to the antennae, thus creating conditions for excitation of BVMSW (similar to [10]).

Measurements were carried out in two stages. At first the optimum regimes for formation and propagation of both SW envelope solitons and SW bullets were determined by means of traditional microwave methods. The parameters used in the present study for the SW envelope soliton formation in a narrow film (SW waveguide) are $f_0 = 8040 MHz$, $H = 2098 Oe$, $\tau = 20 ns$, $P = 350 mW$, and the SW bullets were effectively formed in a wide film sample for $f_0 = 9040 MHz$, $H = 2450 Oe$, $\tau = 29 ns$, $P = 2W$. The calculations based on the BVMSW dispersion equation [12] show, that for the chosen values of f_0 and H the carrier wave-vector in the "soliton" case is $k_{0z} = 70 rad/cm$, while the carrier wave-vector in the "bullet" case is $k_{0z} = 100 rad/cm$. Mainly due to a larger thickness of the wide YIG sample, the observed group velocities of SW bullets were larger than the group velocities of SW solitons, which resulted in different collision times for solitons of $T_{cs} = 165 ns$, and for bullets of $T_{cb} = 120 ns$.

Second, the local intensity of a propagating spin wave pulse (which is proportional to the squared modulus of the local dynamic magnetization in the film, or to the square of the local precession angle) was measured across the sample in steps of $0.1 mm$ by the space- and time-resolved BLS technique [10,13]. For the chosen forward scattering geometry our BLS system was sensitive to spin waves with wave vectors of up to $10^4 rad/cm$. The microwave pulses were applied to the antennae with a repetition frequency of $1 MHz$. At each point a complete time response of the local dynamic magnetization to the propagating spin wave pulse (i.e. the transitional process caused by the propagating SW pulse) was recorded. The complete set of recorded temporal functions was then processed to create two-dimensional maps of the spin wave intensity corresponding each to a given propagation time T.

Fig.1 and Fig.2 show the experimental profiles of two contra-propagating SW packets in a YIG film waveguide and a wide YIG film sample, obtained for different values of the propagation time T as indicated. The upper parts of these figures show two-dimensional SW intensity distributions of the propagating wave packets normalized to the maximum intensity of the largest wave packet, while the lower parts show the cross-section of the intensity distributions for each wave packet taken at its half-maximum. The microstrip antennae, from which the two contra-propagating SW packets have been launched, were oriented along the y-axis, and were situated at $z = 0$ and $z = 8 mm$, respectively. The radiation efficiencies of the left and right input antennae were slightly different, so even though the input microwave pulses supplied to both antennae were identical, the amplitude of the SW packet propagating from the left was about 20% lower than the amplitude of the SW packet propagating from the right.

It is clear from Fig.1a and Fig.2a, that at relatively small propagation times $T = 70 - 80 ns$ the SW packets in the waveguide (Fig. 1a), and in the wide film sample (Fig. 2a) are both two-dimensional, similar in shape, and have similar elliptical cross-sections.

However, in a narrow YIG film waveguide, after some propagation time, the SW packets become quasi-one-dimensional. The packets cross-sections become elongated along the y-axis, and the packets start to occupy almost the full width of the waveguide (Fig.1b). We can assume that at this point quasi-one-dimensional SW envelope solitons are formed from the initially two-dimensional SW packets. At the collision point (Fig.1c) the cross-section of the resulting SW packet is very similar to the cross-sections of the individual SW solitons before the collision. It is also clear from Fig.1d (note, that the vertical scale in Figs. 1d and 2d is 5 times smaller than in the corresponding Figures a, b, and c) that the soliton profiles after the collision are almost the same as before the collision, i.e., the quasi-one-dimensional SW packets in the YIG film waveguide really behave like one-dimensional solitons, and retain their shapes after collisions.

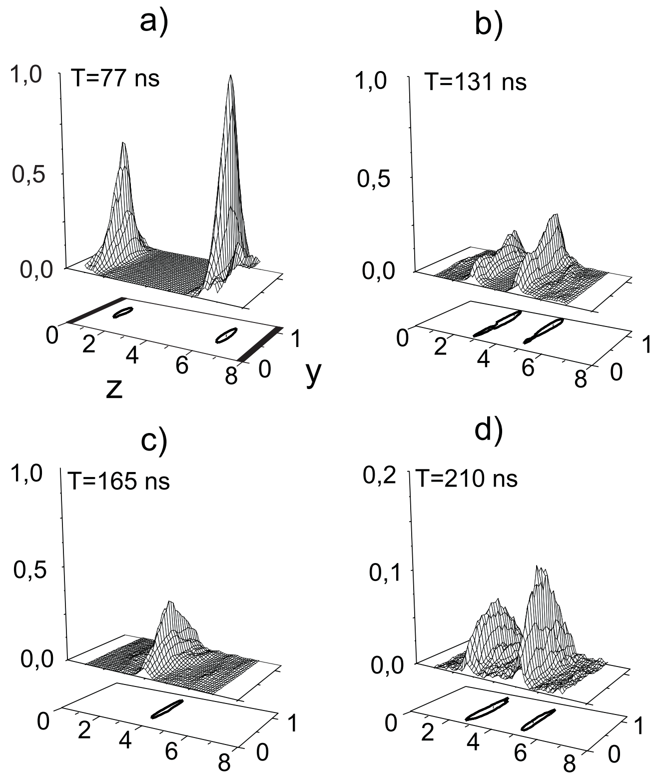

FIG. 1. Formation and collision of quasi-one-dimensional spin wave envelope solitons in a narrow YIG film waveguide. The upper frames show the two-dimensional intensity distributions of the propagating spin wave packets, corresponding to four different values of the propagation time as indicated. The lower frames show the packet's cross-sections at the half-maximum level. The black stripes in Fig.1a show the positions of the antennae.

In a sharp contrast with the previous case, in a wide film sample, where the transverse size of the medium is practically not restricted, strong two-dimensional self-focusing of propagating SW packets takes place. With the increase of propagation time T, the initially elliptical cross-sections of the propagating SW packets become much narrower and almost circular (Fig.2b), which is a well-known feature of wave packets approaching collapse (see e.g. [14]). The collapse, however, is stopped by dissipation, and quasi-stable two-dimensional SW bullets [10] are formed (Fig.2b). At the collision point the increase of intensity in the resulting SW packet, due to the combined effect of the two colliding SW bullets, leads to a catastrophic self-focusing and collapse (Fig.2c), that cannot be anymore stabilized by dissipation. Thus, the equilibrium between self-focusing and dissipation, responsible for the quasi-stability of two-dimensional SW bullets shown in Fig.2b, is broken, and both SW bullets are destroyed in the collision process. The intensity of the SW packets is spread across the whole measurement area (see Fig.2d).

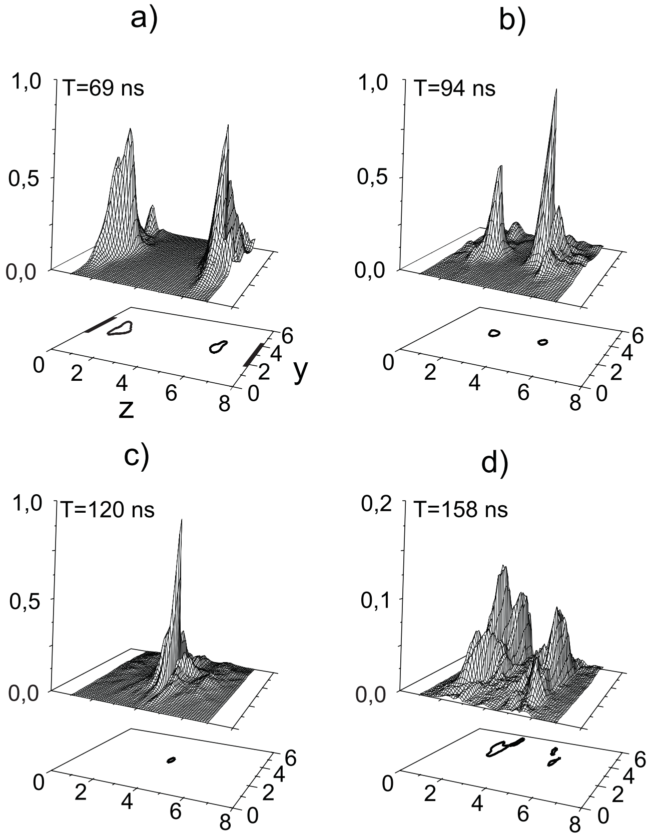

FIG. 2. Formation and collision of two-dimensional self-focused spin wave packets (spin wave bullets) in a wide YIG film sample. The upper frames show the two-dimensional intensity distribution of the propagating wave packets, corresponding to four different values of the propagation time as indicated. The lower frames show the packet's cross-sections at the half-maximum level. The black stripes in Fig.2a show the positions of the antennae.

The qualitative difference between the collision properties of quasi-one-dimensional SW envelope solitons in a waveguide, and two-dimensional SW bullets in a wide film is further illustrated by Fig.3 where the transverse (L_y) and longitudinal (L_z) widths of propagating SW packets are presented as functions of the propagation time T. It is clear, that in a waveguide (Fig. 3 a,b), after the initial period of soliton formation $T < 100ns$, during which the shape of the propagating SW packet becomes elliptical and elongated along the y-direction ($L_y/L_z \approx 2.5$), the spatial sizes of the resulting SW soliton remain practically constant afterwards, and are not significantly affected by the collision with the other soliton, which takes place at $T = T_{cs} = 165ns$. The behavior of the two-dimensional SW packets in a wide film (Fig.3 c,d) is quite different. By the end of the initial period $T < 80ns$, during which a strong two-dimensional self-focusing leading to SW bullet formation takes place, the propagating two-dimensional SW packet (SW bullet) becomes very narrow and almost circular ($L_y/L_z \approx 1$). A subsequent collision with the other bullet at $T = T_{cb} = 120ns$ leads to a dramatic increase of the bullet's sizes along both in-plane directions, and, therefore, to the bullet's destruction at $T > 140ns$ (see Fig.3 c,d).

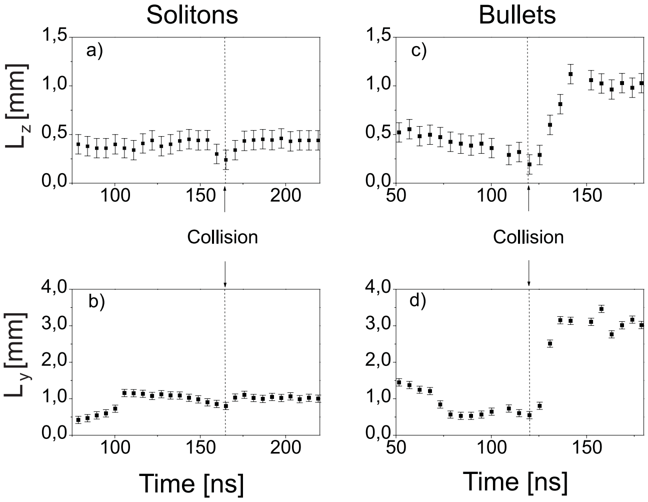

FIG. 3. Longitudinal L_z and transversal L_y width of the spin wave packets propagating from left to right, measured at half-maximum level, as functions of the propagation time T : (a) and (b) - in a waveguide ; (c) and (d) - in a wide film. Note, that the collision events indicated by vertical broken lines are different in the waveguide $T = T_{cs} = 165ns$, and in the wide film $T = T_{cb} = 120ns$.

In conclusion, we have presented first experimental data clearly demonstrating that the space dimensionality is very important for the existence of stable eigenexcitations of nonlinear fields (solitons). In the *quasi-one-dimensional case* of a YIG film waveguide, SW envelope solitons turn out to be stable and practically retain their shapes after collisions with other solitons, even though they are propagating in a medium with relatively large dissipation (typically, around 500 times larger than in optical fibers). On the contrary, in the *two-dimensional* case of a wide film sample the self-focused two-dimensional SW bullets, preserving their shape in a certain range of propagation distances [10], are in reality only quasi-stable, as they suffer almost complete destruction in a head-on collision with another SW bullet.

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