



Fraunhofer Institut
Techno- und
Wirtschaftsmathematik

S. Desmettre, J. Gould, A. Szimayer

Own-company stockholding and
work effort preferences of an
unconstrained executive

© Fraunhofer-Institut für Techno- und Wirtschaftsmathematik ITWM 2008

ISSN 1434-9973

Bericht 146 (2008)

Alle Rechte vorbehalten. Ohne ausdrückliche schriftliche Genehmigung des Herausgebers ist es nicht gestattet, das Buch oder Teile daraus in irgendeiner Form durch Fotokopie, Mikrofilm oder andere Verfahren zu reproduzieren oder in eine für Maschinen, insbesondere Datenverarbeitungsanlagen, verwendbare Sprache zu übertragen. Dasselbe gilt für das Recht der öffentlichen Wiedergabe.

Warennamen werden ohne Gewährleistung der freien Verwendbarkeit benutzt.

Die Veröffentlichungen in der Berichtsreihe des Fraunhofer ITWM können bezogen werden über:

Fraunhofer-Institut für Techno- und
Wirtschaftsmathematik ITWM
Fraunhofer-Platz 1

67663 Kaiserslautern
Germany

Telefon: 06 31/3 16 00-0

Telefax: 06 31/3 16 00-10 99

E-Mail: info@itwm.fraunhofer.de

Internet: www.itwm.fraunhofer.de

Vorwort

Das Tätigkeitsfeld des Fraunhofer-Instituts für Techno- und Wirtschaftsmathematik ITWM umfasst anwendungsnahe Grundlagenforschung, angewandte Forschung sowie Beratung und kundenspezifische Lösungen auf allen Gebieten, die für Techno- und Wirtschaftsmathematik bedeutsam sind.

In der Reihe »Berichte des Fraunhofer ITWM« soll die Arbeit des Instituts kontinuierlich einer interessierten Öffentlichkeit in Industrie, Wirtschaft und Wissenschaft vorgestellt werden. Durch die enge Verzahnung mit dem Fachbereich Mathematik der Universität Kaiserslautern sowie durch zahlreiche Kooperationen mit internationalen Institutionen und Hochschulen in den Bereichen Ausbildung und Forschung ist ein großes Potenzial für Forschungsberichte vorhanden. In die Berichtreihe sollen sowohl hervorragende Diplom- und Projektarbeiten und Dissertationen als auch Forschungsberichte der Institutsmitarbeiter und Institutsgäste zu aktuellen Fragen der Techno- und Wirtschaftsmathematik aufgenommen werden.

Darüber hinaus bietet die Reihe ein Forum für die Berichterstattung über die zahlreichen Kooperationsprojekte des Instituts mit Partnern aus Industrie und Wirtschaft.

Berichterstattung heißt hier Dokumentation des Transfers aktueller Ergebnisse aus mathematischer Forschungs- und Entwicklungsarbeit in industrielle Anwendungen und Softwareprodukte – und umgekehrt, denn Probleme der Praxis generieren neue interessante mathematische Fragestellungen.



Prof. Dr. Dieter Prätzel-Wolters
Institutsleiter

Kaiserslautern, im Juni 2001

Own-Company Stockholding and Work Effort Preferences of an Unconstrained Executive

Sascha Desmettre* John Gould† Alexander Szimayer‡

Abstract

We develop a framework for analyzing an executive's own-company stockholding and work effort preferences. The executive, characterized by risk aversion and work effectiveness parameters, invests his personal wealth without constraint in the financial market, including the stock of his own company whose value he can directly influence with work effort. The executive's utility-maximizing personal investment and work effort strategy is derived in closed-form, and an indifference utility rationale is demonstrated to determine his required compensation. Our results have implications for the practical and theoretical assessment of executive quality and the benefits of performance contracting. Assuming knowledge of the company's non-systematic risk, our executive's unconstrained own-company investment identifies his work effectiveness (i.e. quality), and also reflects work effort that establishes a base-level that performance contracting should seek to exceed.

JEL Classification: M52, G11

Key Words: optimal portfolio choice, executive compensation

*Department of Financial Mathematics, Fraunhofer ITWM, Fraunhofer-Platz 1, 67663 Kaiserslautern, Germany. Email: sascha.desmettre@itwm.fraunhofer.de.

†UWA Business School, The University of Western Australia, Nedlands 6009, Australia. Email: jgould@biz.uwa.edu.au.

‡Department of Financial Mathematics, Fraunhofer ITWM, Fraunhofer-Platz 1, 67663 Kaiserslautern, Germany. Email: alexander.szimayer@itwm.fraunhofer.de.

1 Introduction

Stemming from the agency theory fundamentals of Ross (1973), Jensen and Meckling (1976), Holmstrom (1979) and others, there has been much concern for the ‘incentivization’ link from equity-based executive compensation to corporate financial performance. The associated academic literature is extensive.¹ Counterpoint to past research, we consider the motivation for an executive with unconstrained (unincentivized) compensation to voluntarily performance-link his personal wealth. We develop a model framework that identifies the joint own-company stockholding and work effort strategy of a utility-maximizing executive. The executive’s compensation is assumed to be incorporated into his up-front total personal wealth, which he invests variously in a risk-free money market account, a diversified market portfolio, or his own company’s stock. The executive is able to beneficially influence the value of his company via work effort; he gains utility from the increased value of his direct stockholding (within his overall personal portfolio), but loses utility for his work effort. The executive is characterized by a risk aversion parameter (γ), and two work effectiveness parameters (κ , representing inverse work productivity, and α , representing disutility stress).

A feature of our framework is that the executive’s work effort, specified in terms of two control variables, non-systematic expected return and volatility (μ and σ), can be restated in terms of a single control variable, the non-systematic Sharpe ratio ($\lambda = (\mu - r)/\sigma$, where r is the risk-free rate of return). This reduces the dimension of the problem and introduces a parameterization based on the well-known Sharpe ratio performance measure. The executive’s optimal personal investment and work effort strategy is then derived in closed-form using stochastic control theory and the corresponding Hamilton-Jacobi-Bellman equations. Other technical papers similarly concerned with dynamic principal-agent models include Cadenillas, Cvitanic and Zapatero (2004), Korn and Kraft (2008) and Ou-Yang (2003), for example.

Our closed-form results demonstrate that an executive with superior work effectiveness (i.e. higher quality) will undertake more work effort for his company. Furthermore, depending on any change in the company’s non-systematic volatility associated with the executive’s work effort (i.e. control strategy), due to risk aversion a higher quality executive will not necessarily undertake a higher own-company stockholding. For application to empirical data, our framework allows an executive quality measure to be backed-out from the observed own-company stockholdings of unconstrained executives (assuming knowledge of non-systematic company volatility). Alternatively, with assumption of executive quality and risk aversion, our framework allows identification of the deviation in own-company stockholding that results from constraining an executive with performance contracting.

Freeing executives to self-incentivize may be a reasonable ‘path of least resistance’ in the light of some recent and not so recent research. For example, Dittmann and Maug

¹The summaries of Murphy (1999) and Core, Guay and Larcker (2003) are useful references.

(2007) were unable to rationalize observed executive compensation. Using a ‘standard’ principal-agent efficient contracting model, their analysis indicated that executives should not, in general, be compensated with options, and that it would commonly be optimal for executives to use private savings to purchase additional stock in their own companies. Bettis, Bizjak and Lemmon (2001) found that high-ranking corporate insiders use collars and swaps to cover a significant proportion of their own-company stockholdings, allowing them to unwind the constraint of equity-based compensation. Ross (2004) repudiated the folklore that giving options to agents makes them more willing to take risks (also see Carpenter (2000)). Jensen and Murphy (1990) proposed that private political forces in the managerial labor market constrain pay-performance sensitivity, leading most CEOs to hold trivial fractions of their firms’ stock. To the contrary, Hall and Liebman (1998) and Core and Larcker (2002), for example, found support for a link from equity-based executive compensation to corporate performance.

Whether subject to constrained or unconstrained compensation, an executive’s performance incentive will reflect a total wealth perspective. Ofek and Yermack (2000) found that once managers reach a certain own-company ownership level, they actively rebalance their personal portfolios when awarded equity compensation. Garvey and Milbourn (2003) found that market risk has little effect on the use of stock-based pay for the average executive, suggesting that executives can undo any undesired market exposure from their incentive contracts by adjusting their personal portfolios. We thus maximize our risk averse executive’s utility with respect to total wealth investable across his own company’s stock, a diversified market portfolio and a risk-free money market account. Our approach has parallels with Jin (2002), but uses a continuous-time setting with arguably a more intuitively appealing specification of work effort and its disutility. See also Cvitanic (2008) for a more general continuous time framework emphasizing incentive effects when the executive can hedge equity-based compensation. A natural future extension for our framework is to specify a constrained executive subject to an imposed own-company stockholding representative of performance contracting, and to contrast his work effort strategy with that of our unconstrained executive.

The paper is organized as follows. Section 2 introduces the notation and terminology, and as a first result the optimality problem is reformulated and simplified. In Section 3 the Hamilton-Jacobi-Bellman equation characterizing the utility maximization problem are derived, and a closed form solution is established. The results are illustrated in Section 4. Section 5 concludes.

2 Notation and Set-up

The financial market is defined on a filtered probability space $(\Omega, \mathcal{F}, P, (\mathcal{F}_t)_{t \geq 0})$ satisfying the usual hypothesis and large enough to support two independent standard Brownian motions, $W^P = (W_t^P)_{t \geq 0}$ and $W = (W_t)_{t \geq 0}$. The investment opportunities available to our executive are a risk-free money market account, a diversified market portfolio and his own

company's stocks. The risk-free money market account has the price process $B = (B_t)_{t \geq 0}$, with dynamics

$$dB_t = r B_t dt, \quad B_0 = 1, \quad (2.1)$$

where r is the instantaneous risk-free rate of return, hence $B_t = e^{rt}$. The price process of the market portfolio, $P = (P_t)_{t \geq 0}$, follows the stochastic differential equation (SDE)

$$dP_t = P_t (\mu^P dt + \sigma^P dW_t^P), \quad P_0 \in \mathbb{R}^+, \quad (2.2)$$

where μ^P is the expected return rate of the market portfolio, σ^P is the market portfolio volatility and $W^P = (W_t^P)_{t \geq 0}$ denotes a standard Brownian motion. The company's non-systematic stock price process, $S^{\mu, \sigma} = (S_t^{\mu, \sigma})_{t \geq 0}$, is a controlled diffusion with SDE

$$dS_t^{\mu, \sigma} = S_t^{\mu, \sigma} (\mu_t dt + \sigma_t dW_t), \quad S_0 \in \mathbb{R}^+, \quad (2.3)$$

where μ is the company's expected return rate in excess of the beta-adjusted market portfolio's expected excess return rate (i.e. the expected return compensation for non-systematic risk), and σ is the company's non-systematic volatility, both controlled by the executive. The 'full' stock price process is simply a portfolio combination of P and S dependent on the company's beta.

The executive influences the company's stock price dynamics by choice of the control strategy (μ, σ) , which is specified to be associated with work effort. Value is added if μ is greater than r , indicating excess return compensation for non-systematic risk.² The executive's instantaneous disutility of work effort is represented by $c_t(\mu_t, \sigma_t)$ for control strategy (μ_t, σ_t) at time t . We assume a Markovian disutility rate, i.e., $c_t(\mu_t, \sigma_t) = c(t, v, \mu_t, \sigma_t)$ where $c : [0, T] \times \mathbb{R}^+ \times [r, \infty) \times \mathbb{R}^+ \rightarrow \mathbb{R}_0^+$ is a continuous and suitably differentiable function.

The executive's initial wealth, inclusive of his compensation, is invested in the financial market. Ongoing continuous time portfolio adjustment is assumed to be free of short-selling constraints, and self-financing (i.e. no funds are added to or withdrawn from the executive's portfolio). The portfolio is allocated with fraction $\pi^P = (\pi_t^P)_{t \geq 0}$ invested in the market portfolio, fraction $\pi^S = (\pi_t^S)_{t \geq 0}$ invested in the company's stocks, and the remainder in the risk-free account. For investment strategy $\pi = (\pi^P, \pi^S)$ and initial wealth $V_0 > 0$, the executive's wealth process, $V^\pi = (V_t^\pi)_{t \geq 0}$, is

$$dV_t^\pi = V_t^\pi \left((1 - \pi_t^P - \pi_t^S) dB_t/B_t + \pi_t^P dP_t/P_t + \pi_t^S dS_t^{\mu, \sigma}/S_t^{\mu, \sigma} \right), \quad V_0 > 0, \quad (2.4)$$

The executive is assumed to maximize his terminal utility for time horizon T , subject to some utility function U , which will be specified when deriving closed-form solutions.

Assuming the control of the company's stock price behavior (μ, σ) is determined exogenously, the executive's *optimal investment decision* is then described by

$$\widehat{\Phi}(t, v) = \sup_{\pi \in \Pi(t, v)} \mathbb{E}^{t, v}[U(V_T^\pi)], \quad \text{for } (t, v) \in [0, T] \times \mathbb{R}^+, \quad (2.5)$$

²The control strategy (μ, σ) can be conceptualized as the executive's corporate investment or financing strategy. For example, identifying and initiating positive net present value projects and optimal debt versus equity financing entails work effort that adds value and affects volatility.

where $\Pi(t, v)$ denotes the set of all admissible portfolio strategies π at time t corresponding to the initial wealth v (see for example Korn and Korn (2001)), U is a utility function, and $\mathbb{E}^{t,v}$ denotes the conditional expectation with $V_t = v$; and the exogenously given control (μ, σ) affecting the dynamics of S in (2.3) is suppressed in our notation.

Definition 2.1. *Let $0 \leq t \leq T$, t fixed. Further let (μ, σ) take values in $(r, \infty) \times (0, \infty) \cup \{(r, 0)\}$. By $A(t, v)$ we denote the set of admissible strategies $(\pi, \mu, \sigma) = ((\pi^P, \pi^S), \mu, \sigma)$ corresponding to an initial capital of $v > 0$ at time t , i.e. $\{\mathcal{F}_u; t \leq u \leq T\}$ -predictable processes such that,*

(i) *the wealth equation*

$$dV_u^\pi = V_u^\pi \left((1 - \pi_u^P - \pi_u^S) dB_u/B_u + \pi_u^P dP_u/P_u + \pi_u^S dS_u^{\mu, \sigma}/S_u^{\mu, \sigma} \right), \quad V_t = v,$$

has a unique non-negative solution and satisfies

$$\int_t^T [(V_u^\pi)^2 ((\pi_u^P \sigma^P)^2 + (\pi_u^S \sigma_u)^2)] du < \infty \quad P - a.s.,$$

where (μ, σ) affects V^π via $S^{\mu, \sigma}$,

(ii) *and*

$$\mathbb{E} \left[U(V_T^\pi)^- + \int_t^T c_u(\mu_u, \sigma_u) du \right] < \infty.$$

The *optimal investment and control decision* is then the solution of

$$\Phi(t, v) = \sup_{(\pi, \mu, \sigma) \in A(t, v)} \mathbb{E}^{t,v} \left[U(V_T^\pi) - \int_t^T c_u(\mu_u, \sigma_u) du \right], \quad \text{for } (t, v) \in [0, T] \times \mathbb{R}^+, \quad (2.6)$$

where $\mathbb{E}^{t,v}$ denotes the conditional expectation at t with $V_t = v$, and the utility function U satisfies $U = U^\gamma$ for some $\gamma > 0$. To ensure sensible solutions we require $\mu \geq r$, which effectively bars the executive from destroying company value ($\mu < r$) and potentially profiting by shorting the company's stocks.

2.1 Restating the Set-up

First a decomposition result for the optimal investment and control problem in (2.6) is derived. The original four-dimensional maximization problem can be solved in two steps. The first step is minimizing the disutility rate for a target non-systematic Sharpe ratio $\lambda = (\mu - r)/\sigma$ obtaining $c^*(t, v, \lambda)$. This will be done in Proposition 2.1. Then we will show in Theorem 2.2 that the optimal investment and control problem can then be restated as a maximization problem over the three controls π^P , π^S and λ , where c is replaced by c^* in (2.6).

The following conditions are required for existence and uniqueness of c^* .

Assumption 2.1. The function $c : [0, T] \times \mathbb{R}^+ \times [r, \infty) \times \mathbb{R}^+ \rightarrow \mathbb{R}_0^+$, $(t, v, \mu, \sigma) \mapsto c(t, v, \mu, \sigma)$ satisfies:

(i) c is continuous in t and v , and twice continuously differentiable in μ and σ ;

(ii) Fix $(t, v, \lambda) \in [0, T] \times \mathbb{R}^+ \times \mathbb{R}_0^+$, then

$$\limsup_{\sigma \searrow 0} \lambda \frac{\partial c}{\partial \mu}(t, v, r + \lambda \sigma, \sigma) + \frac{\partial c}{\partial \sigma}(t, v, r + \lambda \sigma, \sigma) \leq 0,$$

and

$$\sup_{\sigma > 0} \lambda \frac{\partial c}{\partial \mu}(t, v, r + \lambda \sigma, \sigma) + \frac{\partial c}{\partial \sigma}(t, v, r + \lambda \sigma, \sigma) > 0.$$

(iii) It holds

$$(\mu - r)^2 \frac{\partial^2 c}{\partial \mu^2} + 2 \sigma (\mu - r) \frac{\partial^2 c}{\partial \mu \partial \sigma} + \sigma^2 \frac{\partial^2 c}{\partial \sigma^2} > 0.$$

(iv) For all (t, v) : $\inf_{\sigma > 0} c(t, v, r, \sigma) = 0$.

In Assumption 2.1, (i) is a natural smoothness condition, (ii) and (iii) are ensuring uniqueness and existence, respectively, of the disutility $c^*(t, v, \lambda)$ depending on the Sharpe ratio λ , and (iv) is a natural norming condition attributing no disutility when no excess return is generated ($\mu = r$) for a specific volatility choice.

A function c which fulfills the conditions of Assumption 2.1 is for example

$$c(t, v, \mu, \sigma) = \kappa \left(\frac{\mu - r}{\sigma} \right)^\alpha + \nu (\sigma - \sigma_0)^2,$$

where $\mu \geq r$, $\sigma > 0$, $\kappa, \nu \geq 0$, $\alpha > 0$ and $\sigma_0 > 0$ is the base-level own-company risk.

Lemma 2.1. Suppose Assumption 2.1 holds, then the minimization problem

$$\min_{\{\sigma > 0: \mu = r + \lambda \sigma\}} c(t, v, \mu, \sigma), \quad \text{for } (t, v, \lambda) \in [0, T] \times \mathbb{R}^+ \times \mathbb{R}_0^+, \quad (2.7)$$

admits a unique solution $\sigma^*(t, v, \lambda)$.

Proof. Fix $(t, v, \lambda) \in [0, T] \times \mathbb{R}^+ \times \mathbb{R}_0^+$ and define the function f by $f(\sigma) = c(t, v, r + \lambda \sigma, \sigma)$, for $\lambda \geq 0$. We need to show that for f a minimizing $\sigma^* = \sigma^*(t, v, \lambda)$ exists and is unique. Computing the first and second derivatives and Assumption 2.1 gives

$$f'(\sigma) = \lambda \frac{\partial c}{\partial \mu}(t, v, r + \lambda \sigma, \sigma) + \frac{\partial c}{\partial \sigma}(t, v, r + \lambda \sigma, \sigma),$$

and

$$f''(\sigma) = \lambda^2 \frac{\partial^2 c}{\partial \mu^2}(t, v, r + \lambda \sigma, \sigma) + 2 \lambda \frac{\partial^2 c}{\partial \sigma \partial \mu}(t, v, r + \lambda \sigma, \sigma) + \frac{\partial^2 c}{\partial \sigma^2}(t, v, r + \lambda \sigma, \sigma),$$

Now f' is continuous differentiable and f'' is continuous by the differentiability assumptions on c . Using elementary calculus rationale, the minimization problem $\min_{\sigma>0} f(\sigma)$ admits a solution if $f'(\sigma^*) = 0$ has a solution and $f''(\sigma^*) > 0$, moreover, the stronger condition f is strictly convex, i.e. $f'' > 0$, implies the solution is a minimizer and unique. Part (iii) of Assumption 2.1 gives the strict convexity of f .

Finally, for $f'(\sigma^*) = 0$ to admit a solution it is sufficient that f' starts below zero, $f'(0+) < 0$, and then the strict convexity implies that f' is strictly increasing. Thus requiring that f' takes on a positive value for some σ ensures the existence of σ with $f'(\sigma^*) = 0$. Assumption 2.1 (ii) implies these conditions. \square

Changing the parameters as described above from π^P , π^S , and (μ, σ) to π^P , π^S , and λ , and replacing c by c^* requires adapting Definition 2.1 to the new setting. Before we present the new framework, observe that the company's non-systematic stock dynamics w.r.t. to λ (and $\sigma^*(\lambda)$) now read:

$$dS_t^\lambda = S_t^\lambda [r dt + \lambda \sigma^*(t, v, \lambda) dt + \sigma^*(t, v, \lambda) dW_t] , \quad S_0 \in \mathbb{R}^+ . \quad (2.8)$$

Definition 2.2. Let $0 \leq t \leq T$, t fixed, and let λ take values in $[0, \infty)$. Define c^* by

$$c^*(t, v, \lambda) := c(t, v, r + \lambda \sigma^*(t, v, \lambda), \sigma^*(t, v, \lambda)) = \min_{\{\sigma>0; \mu=r+\lambda\sigma\}} c(t, v, \mu, \sigma) . \quad (2.9)$$

Then by $A'(t, v)$ we denote the set of admissible strategies $(\pi, \lambda) = ((\pi^P, \pi^S), \lambda)$ corresponding to an initial capital of $v > 0$ at time t , i.e. $\{\mathcal{F}_u; t \leq u \leq T\}$ -predictable processes such that,

(i) the wealth equation

$$dV_u^\pi = V_u^\pi ((1 - \pi_u^P - \pi_u^S) dB_u/B_u + \pi_u^P dP_u/P_u + \pi_u^S dS_u^\lambda/S_u^\lambda) , \quad V_t = v ,$$

has a unique non-negative solution and satisfies

$$\int_t^T [(V_u^\pi)^2 ((\pi_u^P \sigma^P)^2 + (\pi_u^S \sigma_u^*)^2)] du < \infty \quad P - a.s. ,$$

where λ affects V^π via S^λ ,

(ii) and

$$\mathbb{E} \left[U(V_T^\pi)^- + \int_t^T c_u^*(\lambda_u) du \right] < \infty .$$

Theorem 2.2. Suppose (2.6) admits a solution Φ , then it coincides with the value function of the optimal control problem

$$\Phi(t, v) = \sup_{(\pi, \lambda) \in A'(t, v)} \mathbb{E}^{t, v} \left[U(V_T^\pi) - \int_t^T c^*(u, V_u^\pi, \lambda_u) du \right] , \quad \text{for } (t, v) \in [0, T] \times \mathbb{R}^+ , \quad (2.10)$$

where $A'(t, v)$ and c^* are given in Definition 2.2.

Proof. Let

$$J(t, v; \pi, \mu, \sigma) := \mathbb{E}^{t,v} \left[U(V_T^\pi) - \int_t^T c(u, V_u^\pi, \mu(u, V_u^\pi), \sigma(u, V_u^\pi)) du \right]$$

and

$$J'(t, v; \pi, \lambda) := \mathbb{E}^{t,v} \left[U(V_T^\pi) - \int_t^T c^*(u, V_u^\pi, \lambda(u, V_u^\pi)) du \right].$$

The assertion is proven if we show that

$$\sup_{(\pi, \mu, \sigma) \in A(t, v)} J(t, v; \pi, \mu, \sigma) = \sup_{(\pi, \lambda) \in A'(t, v)} J'(t, v; \pi, \lambda),$$

i.e. the performance functionals J and J' admit the same value function $\Phi(t, v)$.

By $c^*(t, v, \lambda) := c(t, v, r + \lambda\sigma^*, \sigma^*) = \min_{\{\sigma > 0; \mu = r + \lambda\sigma\}} c(t, v, \mu, \sigma)$ we have:

$$\begin{aligned} J(t, v; \pi, \mu, \sigma) &= \mathbb{E}^{t,v} \left[U(V_T^\pi) - \int_t^T c(u, V_u^\pi, \mu(u, V_u^\pi), \sigma(u, V_u^\pi)) du \right] \\ &\leq \mathbb{E}^{t,v} \left[U(V_T^\pi) - \int_t^T c^* \left(u, V_u^\pi, \frac{\mu(u, V_u^\pi) - r}{\sigma(u, V_u^\pi)} \right) du \right] = J' \left(t, v; \pi, \frac{\mu - r}{\sigma} \right), \end{aligned}$$

implying

$$\sup_{(\pi, \mu, \sigma) \in A(t, v)} J(t, v; \pi, \mu, \sigma) \leq \sup_{(\pi, \mu, \sigma) \in A(t, v)} J' \left(t, v; \pi, \frac{\mu - r}{\sigma} \right) = \sup_{(\pi, \lambda) \in A'(t, v)} J'(t, v; \pi, \lambda). \quad (*)$$

Now $c^*(t, v, \lambda) := c(t, v, r + \lambda\sigma^*, \sigma^*)$ gives:

$$\begin{aligned} J'(t, v; \pi, \lambda) &= \mathbb{E}^{t,v} \left[U(V_T^\pi) - \int_t^T c^*(u, V_u^\pi, \lambda(u, V_u^\pi)) du \right] \\ &= \mathbb{E}^{t,v} \left[U(V_T^\pi) - \int_t^T c(u, V_u^\pi, r + \lambda\sigma^*, \sigma^*) du \right] = J(t, v; \pi, r + \lambda\sigma^*, \sigma^*), \end{aligned}$$

and then

$$\sup_{(\pi, \lambda) \in A'(t, v)} J'(t, v; \pi, \lambda) = \sup_{(\pi, \lambda) \in A'(t, v)} J(t, v; \pi, r + \lambda\sigma^*, \sigma^*) \leq \sup_{(\pi, \mu, \sigma) \in A(t, v)} J(t, v; \pi, \mu, \sigma). \quad (**)$$

Combining (*) and (**) finishes the proof. \square

3 Optimal Strategies

In this section we will use stochastic control techniques to derive closed-form solutions to our investment and control decision problem in Equation (2.10) for special choices of the utility and disutility function, in particular we derive closed-form solutions for utility

functions with constant relative risk aversion. For the relative risk aversion parameter $\gamma > 0$ the utility function U is:

$$U(v) = \begin{cases} \frac{v^{1-\gamma}}{1-\gamma}, & \text{for } \gamma > 0 \text{ and } \gamma \neq 1 \\ \log(v), & \text{for } \gamma = 1, \end{cases} \quad (3.1)$$

and the cost of effort (or disutility) c^* is assumed to satisfy:

$$c^*(t, v, \lambda) = \kappa v^{1-\gamma} \frac{\lambda^\alpha}{\alpha}, \quad \gamma > 0, \quad (3.2)$$

where $\kappa > 0$ is the inverse work productivity, $\alpha > 2$ the disutility stress, and the scaling factor $v^{1-\gamma}$ is based on a similar formulation for the intertemporal utility from consumption in a constant relative risk aversion setting.

For the remainder of the paper we assume that the control problem (2.10) admits a value function $\Phi \in C^{1,2}$.

To guarantee that the candidates which we will derive for the optimal Sharpe ratio, stockholding strategy and value function are indeed the optimal ones, we have to consider a more restrictive class of admissible strategies:

Definition 3.1. *Let $0 \leq t \leq T$, t fixed, and let λ take values in $[0, \infty)$. Then by $A'_\gamma(t, v)$ we denote the set of admissible strategies $(\pi, \lambda) \in A'(t, v)$, such that*

(i) for $0 < \gamma < 1$:

$$\int_t^T \lambda_u^2 du \leq C < \infty, \text{ for some } C \in \mathbb{R}_0^+, \quad (3.3)$$

(ii) for $\gamma = 1$:

$$\mathbb{E} \left[\int_t^T (\pi_u^P \sigma^P)^2 + (\pi_u^S \sigma_u^*)^2 du \right] < \infty, \quad (3.4)$$

(iii) for $\gamma > 1$:

$$\int_t^T (\pi_u^P \sigma^P)^4 + (\pi_u^S \sigma_u^*)^4 du \leq C_1 < \infty, \text{ for some } C_1 \in \mathbb{R}_0^+, \quad (3.5)$$

$$\int_t^T \pi_u^S \sigma_u^* \lambda_u du \geq C_2 > -\infty, \text{ for some } C_2 \in \mathbb{R}_0^+. \quad (3.6)$$

The optimal investment and control decision then reads:

$$\Phi(t, v) = \sup_{(\pi, \lambda) \in A'_\gamma(t, v)} \mathbb{E}^{t, v} \left[U(V_T^\pi) - \int_t^T c^*(u, V_u^\pi, \lambda_u) du \right], \text{ for } (t, v) \in [0, T] \times \mathbb{R}^+, \quad (3.7)$$

Remark 3.1. *One directly sees that $A'_\gamma(t, v)$ is a subset of $A'(t, v)$. Therefore the results derived in the previous sections remain valid for $A'_\gamma(t, v)$, too.*

3.1 Hamilton-Jacobi-Bellman Equation

Having formulated the optimal investment and control decision problem with respect to the parameter set (π, λ) as in equation (3.7), we can write down the corresponding Hamilton-Jacobi-Bellman equation; thereby note that we formulate this equation w.r.t. general utility functions U and disutility functions c^* :

$$\begin{aligned} 0 &= \sup_{(\pi, \lambda) \in \mathbb{R} \times [0, \infty)} \left[(L^{(\pi, \lambda)} \Phi)(t, v) - c^*(t, v, \lambda) \right], \quad \text{for } (t, v) \in [0, T] \times \mathbb{R}^+, \\ U(v) &= \Phi(T, v), \quad \text{for } v \in \mathbb{R}^+, \end{aligned} \quad (3.8)$$

where the differential operator $L^{(\pi, \lambda)}$ is given by

$$\begin{aligned} (L^{\pi, \lambda} g)(t, v) &= \frac{\partial g}{\partial t}(t, v) + \frac{\partial g}{\partial v}(t, v) v (r + \pi^S \lambda \sigma^*(t, v, \lambda) + \pi^P (\mu^P - r)) \\ &\quad + \frac{1}{2} \frac{\partial^2 g}{\partial v^2}(t, v) v^2 ((\pi^S \sigma^*(t, v, \lambda))^2 + (\pi^P \sigma^P)^2). \end{aligned} \quad (3.9)$$

Potential maximizers π^{P^*} , π^{S^*} and λ^* of the HJB (3.8) can be calculated by establishing the first order conditions:

$$\begin{aligned} \pi^{P^*}(t, v) &= -\frac{(\mu^P - r) \Phi_v(t, v)}{v(\sigma^P)^2 \Phi_{vv}(t, v)}, \\ \pi^{S^*}(t, v) &= -\frac{\lambda^*(t, v) \Phi_v(t, v)}{v\sigma^*(t, v, \lambda^*(t, v)) \Phi_{vv}(t, v)}, \end{aligned} \quad (3.10)$$

where λ^* is the solution of the implicit equation

$$\lambda \frac{\Phi_v^2(t, v)}{\Phi_{vv}(t, v)} + c_\lambda^*(t, v, \lambda) = 0 \quad \text{for all } (t, v) \in [0, T] \times \mathbb{R}^+, \quad (3.11)$$

where we have already used representation (3.10) to simplify the equation.

The executive's optimal wealth allocation to his own company depends on his stock price dynamics control decision, $\lambda = \lambda^*$, whereas allocation to the market portfolio does not. However, recalling that the own-company allocation is with respect to the company's non-systematic stock price process, implicit to this result is that the executive's actual market portfolio allocation is the net of his 'full' market portfolio allocation (π^P) and the systematic exposure of his own-company stockholding dependent on the company's beta.

Substituting the maximizers (3.10) in the HJB (3.8) then yields:

$$\Phi_t(t, v) + \Phi_v(t, v) v r - \frac{1}{2} (\lambda^*(t, v))^2 \frac{\Phi_v^2(t, v)}{\Phi_{vv}(t, v)} - \frac{1}{2} (\lambda_P)^2 \frac{\Phi_v^2(t, v)}{\Phi_{vv}(t, v)} - c^*(t, v, \lambda^*(t, v)) = 0, \quad (3.12)$$

where $\lambda_P := \frac{\mu^P - r}{\sigma^P}$.

In the following we aim at solving Equation (3.12) for the choices (3.1) and (3.2) of the utility and disutility functions.

3.2 Closed-Form Solutions

In this section we present the closed-form solutions to our control problem (3.7) for the utility and disutility functions specified in (3.1) and (3.2).

Theorem 3.1 (The power utility case: $\gamma > 0$ and $\gamma \neq 1$). *The full solution of the maximization problem (3.7) can then be summarized by the strategy*

$$\lambda^*(t, v) = \left(\frac{1}{\kappa \gamma} f(t) \right)^{\frac{1}{\alpha-2}}, \quad \pi^{P^*}(t, v) = \frac{\mu^P - r}{\gamma (\sigma^P)^2}, \quad \pi^{S^*}(t, v) = \frac{\lambda^*(t, v)}{\gamma \sigma^*(t, v, \lambda^*(t, v))}, \quad (3.13)$$

and value function

$$\Phi(t, v) = \frac{v^{1-\gamma}}{1-\gamma} f(t), \quad (3.14)$$

where

$$f(t) = e^{(1-\gamma) \left(r + \frac{1}{2} \frac{\lambda_P^2}{\gamma} \right) (T-t)} \left(1 - \frac{(\alpha-2) \left(\frac{1}{\kappa \gamma} \right)^{\frac{2}{\alpha-2}} \left(e^{\frac{1-\gamma}{\alpha-2} \left(2r + \frac{\lambda_P^2}{\gamma} \right) (T-t)} - 1 \right)}{\alpha (2\gamma r + \lambda_P^2)} \right)^{-\frac{\alpha-2}{2}}. \quad (3.15)$$

Proof. First observe that a function F of the form $F(\lambda) = a \lambda^2 - b \lambda^\alpha$, $\lambda \geq 0$, for given constant $a, b > 0$ and $\alpha > 2$, has a unique maximizer λ^* and maximized value $F(\lambda^*)$ given by

$$\lambda^* = \left(\frac{2a}{\alpha b} \right)^{\frac{1}{\alpha-2}}, \quad \text{and} \quad F(\lambda^*) = (\alpha-2) \alpha^{-\frac{\alpha}{\alpha-2}} 2^{\frac{2}{\alpha-2}} a^{\frac{\alpha}{\alpha-2}} b^{-\frac{2}{\alpha-2}}. \quad (3.16)$$

Using this insight the first order condition for λ^* in Equation (3.11) is now solved. Set

$$a = \frac{1}{2} \frac{\Phi_v^2}{-\Phi_{vv}}, \quad \text{and} \quad b = \frac{\kappa}{\alpha} v^{1-\gamma},$$

then Equation (3.16) gives

$$\lambda^* = \left(\frac{1}{\kappa v^{1-\gamma}} \frac{\Phi_v^2}{-\Phi_{vv}} \right)^{\frac{1}{\alpha-2}}, \quad \text{and} \quad F(\lambda^*) = \frac{\alpha-2}{2\alpha} (\kappa v^{1-\gamma})^{-\frac{2}{\alpha-2}} \left(\frac{\Phi_v^2}{-\Phi_{vv}} \right)^{\frac{\alpha}{\alpha-2}}.$$

Now Equation (3.12) reads

$$0 = \Phi_t + \Phi_v v r + \frac{1}{2} \frac{\Phi_v^2}{-\Phi_{vv}} \left(\frac{\mu^P - r}{\sigma^P} \right)^2 + \frac{\alpha-2}{2\alpha} (\kappa v^{1-\gamma})^{-\frac{2}{\alpha-2}} \left(\frac{\Phi_v^2}{-\Phi_{vv}} \right)^{\frac{\alpha}{\alpha-2}}. \quad (3.17)$$

Using the separation ansatz $\Phi(t, v) = f(t) \frac{v^{1-\gamma}}{1-\gamma}$ results in

$$\Phi_t = \dot{f} \frac{v^{1-\gamma}}{1-\gamma}, \quad \Phi_v = f v^{-\gamma}, \quad \Phi_{vv} = -\gamma f v^{-\gamma-1}, \quad \text{and} \quad f(T) = 1. \quad (3.18)$$

Equation (3.17) then becomes

$$0 = \dot{f} \frac{v^{1-\gamma}}{1-\gamma} + f v^{1-\gamma} r + \frac{1}{2} \frac{f v^{1-\gamma}}{\gamma} \left(\frac{\mu^P - r}{\sigma^P} \right)^2 + \frac{\alpha - 2}{2\alpha} (\kappa v^{1-\gamma})^{-\frac{2}{\alpha-2}} \left(\frac{f v^{1-\gamma}}{\gamma} \right)^{\frac{\alpha}{\alpha-2}}.$$

Dividing by $\frac{v^{1-\gamma}}{1-\gamma}$ and recalling $\lambda_P = (\mu^P - r)/\sigma^P$ gives

$$\dot{f} = f \left[-(1-\gamma) \left(r + \frac{1}{2} \frac{\lambda_P^2}{\gamma} \right) \right] + f^{\frac{\alpha}{\alpha-2}} \left[-(1-\gamma) \frac{\kappa}{2} \frac{\alpha-2}{\alpha} \left(\frac{1}{\kappa \gamma} \right)^{\frac{\alpha}{\alpha-2}} \right]. \quad (3.19)$$

This is a Bernoulli ODE of the form $\dot{f} = a_1 f + a_n f^n$, with solution

$$f(t)^{1-n} = C e^{G(t)} + (1-n) e^{G(t)} \int_0^t e^{-G(s)} a_n ds,$$

where $G(t) = (1-n) \int_0^t a_1(s) ds$ and C an arbitrary constant. In our setting we have $n = \frac{\alpha}{\alpha-2}$ and

$$(1-n) = \frac{-2}{\alpha-2}, \quad a_1 = -(1-\gamma) \left(r + \frac{1}{2} \frac{\lambda_P^2}{\gamma} \right), \quad \text{and} \quad a_n = -(1-\gamma) \frac{\kappa}{2} \frac{\alpha-2}{\alpha} \left(\frac{1}{\kappa \gamma} \right)^{\frac{\alpha}{\alpha-2}}.$$

The formal solution $f(t)^{1-n}$ is explicitly calculated in three steps. First, compute

$$G(t) = -\frac{2a_1 t}{\alpha-2}, \quad \text{and} \quad \int_0^t e^{-G(s)} a_n(s) ds = \frac{\alpha-2}{2} \frac{a_n}{a_1} \left(e^{\frac{2a_1 t}{\alpha-2}} - 1 \right),$$

then

$$f(t) = e^{a_1 t} \left(C - \frac{a_n}{a_1} \left(e^{\frac{2a_1 t}{\alpha-2}} - 1 \right) \right)^{-\frac{\alpha-2}{2}}.$$

Finally, solve for C by using $f(T) = 1$ and

$$C = e^{\frac{2a_1 T}{\alpha-2}} + \frac{a_n}{a_1} \left(e^{\frac{2a_1 T}{\alpha-2}} - 1 \right).$$

Note also that $f(0) = C^{-\frac{\alpha-2}{2}}$. Now

$$f(t) = e^{-a_1(T-t)} \left(1 - \frac{a_n}{a_1} \left(e^{-\frac{2a_1}{\alpha-2}(T-t)} - 1 \right) \right)^{-\frac{\alpha-2}{2}}.$$

Plugging in a_1 and a_n then yields the result for $f(t)$. Using $\frac{\Phi_v}{\Phi_{vv}} = -\frac{v}{\gamma}$ and the first order condition in Equation (3.10) we obtain the claimed optimal strategies λ^* , π^{P^*} and π^{S^*} . Note that our claimed optimal controls are deterministic and continuous on a compact support, so there are uniformly bounded, which shows that $(\pi^{S^*}, \pi^{P^*}, \lambda^*) \in A'_\gamma(t, v)$. \square

The following theorem gives the results for the log-utility case.

Theorem 3.2 (The log-utility case, $\gamma = 1$). *The full solution of the maximization problem (3.7) can then be summarized by the strategy*

$$\lambda^*(t, v) = \kappa^{-\frac{1}{\alpha-2}}, \quad \pi^{P^*}(t, v) = \frac{\mu^P - r}{(\sigma^P)^2}, \quad \text{and} \quad \pi^{S^*}(t, v) = \frac{\lambda^*(t, v)}{\sigma^*(t, v, \lambda^*(t, v))}, \quad (3.20)$$

and value function

$$\Phi(t, v) = \log(v) + \left[r + \frac{1}{2} \left(\frac{\mu^P - r}{\sigma^P} \right)^2 + \frac{\alpha - 2}{2\alpha} \kappa^{-\frac{2}{\alpha-2}} \right] (T - t). \quad (3.21)$$

Proof. As in the power-utility case, first the implicit first order condition for λ^* in Equation 3.11 is made explicit. This time set

$$a = \frac{1}{2} \frac{\Phi_v^2}{-\Phi_{vv}}, \quad \text{and} \quad b = \frac{\kappa}{\alpha},$$

then Equation 3.16 gives

$$\lambda^* = \left(\frac{1}{\kappa} \frac{\Phi_v^2}{-\Phi_{vv}} \right)^{\frac{1}{\alpha-2}}, \quad \text{and} \quad F(\lambda^*) = \frac{\alpha - 2}{2\alpha} \kappa^{-\frac{2}{\alpha-2}} \left(\frac{\Phi_v^2}{-\Phi_{vv}} \right)^{\frac{\alpha}{\alpha-2}}.$$

The PDE for log-utility reads now

$$0 = \Phi_t + \Phi_v v r + \frac{1}{2} \frac{\Phi_v^2}{-\Phi_{vv}} \left(\frac{\mu^P - r}{\sigma^P} \right)^2 + \frac{\alpha - 2}{2\alpha} \kappa^{-\frac{2}{\alpha-2}} \left(\frac{\Phi_v^2}{-\Phi_{vv}} \right)^{\frac{\alpha}{\alpha-2}}. \quad (3.22)$$

Using the ansatz $\Phi(t, v) = \log(v) + \varphi(T - t)$ results in

$$\Phi_t = -\varphi, \quad \Phi_v = \frac{1}{v}, \quad \Phi_{vv} = -\frac{1}{v^2}, \quad \text{and} \quad \Phi(T, v) = \log(v) = U(v).$$

Then Equation (3.22) reduces to

$$\varphi = r + \frac{1}{2} \left(\frac{\mu^P - r}{\sigma^P} \right)^2 + \frac{\alpha - 2}{2\alpha} \kappa^{-\frac{2}{\alpha-2}}.$$

Finally noting $\Phi_v^2/\Phi_{vv} = -1$ and recalling the first order condition for the portfolio strategy in Equation (3.10) establish the claimed optimal controls. Again, our claimed controls are deterministic and continuous on a compact support, so there are uniformly bounded, which then proves that $(\pi^{S^*}, \pi^{P^*}, \lambda^*) \in A'_\gamma(t, v)$. Note that we also obtain the form of the optimal strategies by formally setting $\gamma = 1$ in Theorem 3.1. \square

3.3 Verification Theorem

The solutions of the maximization problems given in Theorem 3.1 and Theorem 3.2 are candidates for the optimal work effort and own-company stockholding of the control problem in (3.7). In this section we will verify that under sufficient assumptions these are indeed optimal.

Theorem 3.3 (Verification Result). *Let $\kappa > 0$ and $\alpha > 2$. Assume that the investor has a utility and disutility function of the form given in (3.1) and (3.2). Then the candidates given in (3.13) - (3.15) are the optimal Sharpe ratio and stockholding strategy and value function of the optimal control problem (3.7) for the case $\gamma > 0$ and $\gamma \neq 1$ and the candidates given in (3.20) and (3.21) are the optimal Sharpe ratio and stockholding strategy and value function of the optimal control problem (3.7) for the case $\gamma = 1$.*

Proof. Define the performance functional of our optimal investment and control decision by

$$J'(t, v; \pi, \lambda) := \mathbb{E}^{t,v} \left[U(V_T^\pi) - \int_t^T c_u^*(\lambda_u) du \right].$$

Our candidates are optimal if we have

$$J'(t, v; \pi^*, \lambda^*) = \Phi(t, v) \text{ and } J'(t, v; \pi, \lambda) \leq \Phi(t, v), \text{ for all } (\pi, \lambda) \in A'_\gamma(t, v).$$

Part 1: $\gamma > 0$ and $\gamma \neq 1$.

Let $(\pi, \lambda) \in A'_\gamma(t, v)$. Since $\Phi \in C^{1,2}$, we obtain by Ito's formula:

$$\begin{aligned} \Phi(T, V_T^\pi) - \int_t^T \kappa (V_u^\pi)^{1-\gamma} \frac{\lambda_u^\alpha}{\alpha} du &= \Phi(t, v) + \int_t^T \Phi_t(u, V_u^\pi) du \\ &+ \int_t^T \Phi_v(u, V_u^\pi) V_u^\pi [(1 - \pi_u^P - \pi_u^S) r + \pi_u^P \mu^P + \pi_u^S (r + \lambda \sigma_u^*)] du \\ &+ \int_t^T \Phi_v(u, V_u^\pi) V_u^\pi \pi_u^P \sigma^P dW_u^P + \int_t^T \Phi_v(u, V_u^\pi) V_u^\pi \pi_u^S \sigma_u^* dW_u \\ &+ 1/2 \int_t^T \Phi_{vv}(u, V_u^\pi) (V_u^\pi)^2 [(\pi_u^P \sigma^P)^2 + (\pi_u^S \sigma_u^*)^2] du - \int_t^T \kappa (V_u^\pi)^{1-\gamma} \frac{\lambda_u^\alpha}{\alpha} du \\ &= \Phi(t, v) + \int_t^T \left\{ \Phi_t(u, V_u^\pi) + \Phi_v(u, V_u^\pi) V_u^\pi [r + \pi_u^S \lambda \sigma_u^* + \pi_u^P (\mu^P - r)] \right. \\ &\quad \left. + 1/2 \Phi_{vv}(u, V_u^\pi) (V_u^\pi)^2 [(\pi_u^P \sigma^P)^2 + (\pi_u^S \sigma_u^*)^2] - \kappa (V_u^\pi)^{1-\gamma} \frac{\lambda_u^\alpha}{\alpha} \right\} du \\ &+ \int_t^T \Phi_v(u, V_u^\pi) V_u^\pi \pi_u^P \sigma^P dW_u^P + \int_t^T \Phi_v(u, V_u^\pi) V_u^\pi \pi_u^S \sigma_u^* dW_u. \end{aligned} \quad (3.23)$$

For the optimality candidates given in (3.13 - 3.15) we have

$$\mathbb{E}^{t,v} \left[\int_t^T \Phi_v(u, V_u^{\pi^*}) V_u^{\pi^*} \pi_u^{P^*} \sigma^P dW_u^P + \int_t^T \Phi_v(u, V_u^{\pi^*}) V_u^{\pi^*} \pi_u^{S^*} \sigma_u^* dW_u \right] = 0. \quad (3.24)$$

To verify Equation (3.24) it is sufficient to prove the square-integrability condition

$$\mathbb{E} \left[\int_t^T (\Phi_v(u, V_u^{\pi^*}) V_u^{\pi^*} [\pi_u^{P^*} \sigma^P + \pi_u^{S^*} \sigma_u^*])^2 du \right] < \infty. \quad (*)$$

Now plugging in the candidates from (3.13 - 3.15) yields

$$\Phi_v(u, V_u^{\pi^*}) V_u^{\pi^*} [\pi_u^{P^*} \sigma^P + \pi_u^{S^*} \sigma_u^*] = \frac{(V_u^{\pi^*})^{1-\gamma}}{\gamma} \left[\frac{\mu^P - r}{\sigma^P} + \left(\frac{1}{\kappa\gamma} f(u) \right)^{\frac{1}{\alpha-2}} \right]. \quad (**)$$

The RHS of (**) is $(V_u^{\pi^*})^{1-\gamma}$ times a deterministic and continuous function on the compact set $[0, T]$. The deterministic part is therefore uniformly bounded and additionally, $V_u^{\pi^*}$ satisfies the wealth equation

$$dV_t^{\pi^*} = V_t^{\pi^*} [r dt + \lambda_P^2/\gamma dt + (\lambda^*(t, V_t^{\pi^*}))^2/\gamma dt + \lambda_P/\gamma dW_t^P + \lambda^*(t, V_t^{\pi^*})/\gamma dW_t].$$

Recalling that $\lambda^*(t, v)$ is a deterministic function in t and does further not depend on v , we see that $V_t^{\pi^*}$ follows a log-normal distribution for all $t \geq 0$ with parameters being uniformly bounded for all $t \in [0, T]$. Since all moments of a log-normally distributed random variable exist, (*) holds proving (3.24).

Additionally, Φ satisfies the HJB equation (3.8), i.e. for $(\pi, \lambda) = (\pi^*, \lambda^*)$ and the choice (3.2) of the disutility function we have:

$$\begin{aligned} \Phi_t(u, V_u^{\pi^*}) + \Phi_v(u, V_u^{\pi^*}) V_u^{\pi^*} [r + \pi_u^{S^*} \lambda^* \sigma_u^* + \pi_u^{P^*} (\mu^P - r)] \\ + 1/2 \Phi_{vv}(u, V_u^{\pi^*}) [(V_u^{\pi^*} \pi^{P^*} \sigma^P)^2 + (V_u^{\pi^*} \pi_u^{S^*} \sigma_u^*)^2] - \kappa (V_u^{\pi^*})^{1-\gamma} \frac{(\lambda_u^*)^\alpha}{\alpha} = 0. \end{aligned}$$

For $(\pi, \lambda) = (\pi^*, \lambda^*)$, the expectation of equation (3.23) using that $\Phi(T, v) = v^{1-\gamma}/(1-\gamma)$ is:

$$\mathbb{E}^{t,v} \left[\frac{(V_T^{\pi^*})^{1-\gamma}}{1-\gamma} \right] - \mathbb{E}^{t,v} \left[\int_t^T \kappa (V_u^{\pi^*})^{1-\gamma} \frac{(\lambda_u^*)^\alpha}{\alpha} du \right] = J'(t, v; \pi^*, \lambda^*) = \Phi(t, v).$$

The optimality of our candidates is finally shown if we have for all $(\pi, \lambda) \in A'_\gamma(t, v)$:

$$\mathbb{E}^{t,v} \left[\frac{(V_T^\pi)^{1-\gamma}}{1-\gamma} \right] - \mathbb{E}^{t,v} \left[\int_t^T \kappa (V_u^\pi)^{1-\gamma} \frac{(\lambda_u)^\alpha}{\alpha} du \right] = J'(t, v; \pi, \lambda) \leq \Phi(t, v). \quad (3.25)$$

Also, since Φ satisfies the HJB equation (3.8), we get for all $(\pi, \lambda) \in A'_\gamma(t, v)$:

$$\begin{aligned} \Phi_t(u, V_u^\pi) + \Phi_v(u, V_u^\pi) V_u^\pi [r + \pi_u^S \lambda \sigma_u^* + \pi_u^P (\mu^P - r)] \\ + 1/2 \Phi_{vv}(u, V_u^\pi) [(V_u^\pi \pi^P \sigma^P)^2 + (V_u^\pi \pi_u^S \sigma_u^*)^2] - \kappa (V_u^\pi)^{1-\gamma} \frac{(\lambda_u)^\alpha}{\alpha} \leq 0. \end{aligned}$$

Substituting this in Equation (3.23) and recalling that

$$\Phi_t(t, v) = \dot{f}(t) \frac{v^{1-\gamma}}{1-\gamma}, \quad \Phi_v(t, v) = f(t) v^{-\gamma}, \quad \Phi_{vv}(t, v) = -\gamma f(t) v^{-\gamma-1} \quad (3.26)$$

we get:

$$\begin{aligned} \Phi(T, V_T^\pi) - \int_t^T \kappa (V_u^\pi)^{1-\gamma} \frac{\lambda_u^\alpha}{\alpha} du \\ \leq \Phi(t, v) + \underbrace{\int_t^T (V_u^\pi)^{1-\gamma} f(u) \pi_u^P \sigma^P dW_u^P + \int_t^T (V_u^\pi)^{1-\gamma} f(u) \pi_u^S \sigma_u^* dW_u}_{=: M_T^t}. \end{aligned} \quad (3.27)$$

Part 1.1: $0 < \gamma < 1$.

To verify equation (3.25) for the case $0 < \gamma < 1$ we will show that the local martingale M_T^t is a supermartingale. Applying again Ito's formula and using (3.26) yields:

$$\begin{aligned}
\Phi(T, V_T^\pi) &= \Phi(t, v) + \int_t^T \dot{f}(u) \frac{(V_u^\pi)^{1-\gamma}}{1-\gamma} du + \int_t^T f(u) (V_u^\pi)^{1-\gamma} [r + \pi_u^P (\mu^P - r) \\
&\quad + \pi_u^S \lambda \sigma_u^*] du + \int_t^T (V_u^\pi)^{1-\gamma} \pi_u^P \sigma^P dW_u^P + \int_t^T (V_u^\pi)^{1-\gamma} \pi_u^S \sigma_u^* dW_u \\
&\quad - 1/2 \int_t^T \gamma f(u) (V_u^\pi)^{1-\gamma} [(\pi_u^P \sigma^P)^2 + (\pi_u^S \sigma_u^*)^2] du \\
&= \Phi(t, v) + M_T^t + \int_t^T \frac{(V_u^\pi)^{1-\gamma}}{1-\gamma} \left\{ \dot{f}(u) + f(u) [(1-\gamma) (r + \pi_u^P \lambda \sigma^P + \pi_u^S \lambda \sigma_u^*) \right. \\
&\quad \left. - \frac{\gamma}{2} (1-\gamma) ((\pi_u^P \sigma^P)^2 + (\pi_u^S \sigma_u^*)^2)] \right\} du.
\end{aligned}$$

Now recalling that

$$\dot{f} = f \left[-(1-\gamma) \left(r + \frac{1}{2} \frac{\lambda_P^2}{\gamma} \right) \right] + f^{\frac{\alpha}{\alpha-2}} \left[-(1-\gamma) \frac{\kappa}{2} \frac{\alpha-2}{\alpha} \left(\frac{1}{\kappa \gamma} \right)^{\frac{\alpha}{\alpha-2}} \right]$$

and keeping in mind that $\lambda_u^* = \left(\frac{1}{\kappa \gamma} f(u) \right)^{\frac{1}{\alpha-2}}$ we get:

$$\begin{aligned}
\Phi(T, V_T^\pi) &= \Phi(t, v) + M_T^t + \int_t^T (V_u^\pi)^{1-\gamma} f(u) \left[-\frac{1}{2\gamma} \lambda_P^2 - \frac{1}{2\gamma} (\lambda_u^*)^2 \frac{\alpha-2}{\alpha} + \pi_u^P \lambda_P \sigma^P \right. \\
&\quad \left. + \pi_u^S \lambda_u \sigma_u^* - 1/2 \gamma (\pi_u^P \sigma^P)^2 - 1/2 \gamma (\pi_u^S \sigma_u^*)^2 \right] du.
\end{aligned}$$

Some side calculations including completing the square then yield:

$$\begin{aligned}
M_T^t &= \underbrace{\Phi(T, V_T^\pi)}_{(1)} - \underbrace{\Phi(t, v)}_{(2)} + \underbrace{\frac{\lambda_P^2}{2\gamma} \int_t^T (V_u^\pi)^{1-\gamma} f(u) du}_{\geq 0} + \underbrace{\frac{1}{2\gamma} \frac{\alpha-2}{\alpha} \int_t^T (V_u^\pi)^{1-\gamma} f(u) (\lambda_u^*)^2 du}_{\geq 0} \\
&\quad + \underbrace{\frac{\gamma}{2} \int_t^T (\sigma_u^*)^2 \left(\pi_u^S - \frac{\lambda_u}{\gamma \sigma_u^*} \right)^2 + \sigma_P^2 \left(\pi_u^P - \frac{\lambda_P}{\gamma \sigma_P} \right)^2 du}_{\geq 0} - \underbrace{\frac{1}{2\gamma} \int_t^T \lambda_u^2 du}_{(3)} - \frac{\lambda_P^2}{2\gamma} (T-t).
\end{aligned}$$

For $0 < \gamma < 1$, we have that (1) > 0 , (2) is \mathcal{F}_t -measurable and thus deterministic at time t . Due to condition (3.3), (3) is \mathcal{F}_t -measurable too for all t and bounded by the real constant C . Thus, M_T^t is a local martingale which is bounded from below, i.e.

$$M_T^t \geq -\Phi(t, v) - \frac{1}{2\gamma} (C + \lambda_P^2 (T-t)).$$

This implies that M_T^t is a supermartingale and therefore equation (3.27) simplifies to

$$\Phi(T, V_T^\pi) - \int_t^T \kappa (V_u^\pi)^{1-\gamma} \frac{\lambda_u^\alpha}{\alpha} du \leq \Phi(t, v).$$

Taking the expectation on both sides then yields equation (3.25) which finishes the proof for the case $0 < \gamma < 1$.

Part 1.2: $\gamma > 1$.

To verify equation (3.25) for the case $\gamma > 1$ we will impose conditions under which the local martingale M_T^t is a martingale. The straightforward condition for this is

$$\mathbb{E} \left[\int_t^T (V_u^\pi)^{2(1-\gamma)} f^2(u) ((\pi_u \sigma^P)^2 + (\sigma_u^* \pi_u^S)^2) du \right] < \infty. \quad (3.28)$$

But this condition is not handsome enough for our purposes. In what follows we will derive conditions which are independent from the wealth process.

For the integrand of (3.28) the following estimate is valid:

$$(V_u^\pi)^{2(1-\gamma)} f^2(u) ((\pi_u \sigma^P)^2 + (\sigma_u^* \pi_u^S)^2) \leq \underbrace{\frac{1}{2} f^4(u) (V_u^\pi)^{4(1-\gamma)}}_{(1)} + \underbrace{\frac{1}{2} ((\pi_u \sigma^P)^2 + (\sigma_u^* \pi_u^S)^2)^2}_{(2)}$$

From this equation one directly obtains that condition (3.5) in Def. 3.1 ensures that

$$\frac{1}{2} \mathbb{E} \left[\int_t^T ((\pi_u \sigma^P)^2 + (\sigma_u^* \pi_u^S)^2)^2 du \right] < \infty. \quad (3.29)$$

Note that in expression (1), f is a deterministic function on the compact set $[0, T]$ and therefore $\sup_{t \leq u \leq T} f^4(u) < \infty$. So what is left, is to ensure that

$$\frac{1}{2} \mathbb{E} \left[\int_t^T (V_u^\pi)^{4(1-\gamma)} du \right] < \infty. \quad (3.30)$$

The solution of the wealth equation (2.4) expressed w.r.t. to the parameter λ applying variation of constants is:

$$V_t^\pi = V_0^\pi e^{rt + \int_0^t \pi_u^P \lambda^P \sigma^P + \pi_u^S \lambda_u \sigma_u^* du} e^{L_t - \frac{1}{2} \langle L \rangle_t},$$

where $L_t = \int_0^t \pi_u^P \sigma^P dW_u^P + \int_0^t \pi_u^S \sigma_u^* dW_u$ and $\langle L \rangle_t = \int_0^t (\pi_u^P \sigma^P)^2 + (\pi_u^S \sigma_u^*)^2 du$.

Using this we have that

$$\begin{aligned} (V_t^\pi)^{4(1-\gamma)} &= (V_0^\pi)^{4(1-\gamma)} e^{4(1-\gamma)[rt + \int_0^t \pi_u^P \lambda^P \sigma^P + \pi_u^S \lambda_u \sigma_u^* du]} e^{4(1-\gamma)L_t - \frac{1}{2} 4(1-\gamma) \langle L \rangle_t} \\ &= (V_0^\pi)^{4(1-\gamma)} e^{4(1-\gamma)[rt + \int_0^t \pi_u^P \lambda^P \sigma^P + \pi_u^S \lambda_u \sigma_u^* du]} \\ &\quad \cdot e^{4(1-\gamma)L_t - \frac{1}{2} 16(1-\gamma)^2 \langle L \rangle_t} e^{\frac{1}{2} 16(1-\gamma)^2 \langle L \rangle_t - \frac{1}{2} 4(1-\gamma) \langle L \rangle_t} \\ &= (V_0^\pi)^{4(1-\gamma)} e^{4(1-\gamma)L_t - \frac{1}{2} 16(1-\gamma)^2 \langle L \rangle_t} e^{4(1-\gamma)[(2(1-\gamma) - \frac{1}{2}) + rt + \int_0^t \pi_u^P \lambda^P \sigma^P + \pi_u^S \lambda_u \sigma_u^* du]}. \end{aligned}$$

Condition (3.30) is fulfilled, if for example we have that

$$Z_t := e^{4(1-\gamma)L_t - \frac{1}{2} 16(1-\gamma)^2 \langle L \rangle_t} \in L^2(P),$$

and that

$$R_t := e^{4(1-\gamma)\left[(2(1-\gamma)-\frac{1}{2})+rt+\int_0^t \pi_u^P \lambda^P \sigma^P + \pi_u^S \lambda_u \sigma_u^* du\right]} \in L^2(P).$$

We consider the square of Z_t :

$$\begin{aligned} Z_t^2 &= e^{8(1-\gamma)L_t - \frac{1}{2} 32(1-\gamma)^2 \langle L \rangle_t} \\ &= e^{8(1-\gamma)L_t - 64(1-\gamma)^2 \langle L \rangle_t} e^{48(1-\gamma)^2 \langle L \rangle_t} \\ &\leq \underbrace{\frac{1}{2} e^{16(1-\gamma)L_t - \frac{1}{2} 256(1-\gamma)^2 \langle L \rangle_t}}_{(1)} + \underbrace{\frac{1}{2} e^{96(1-\gamma)^2 \langle L \rangle_t}}_{(2)}. \end{aligned}$$

Condition (3.5) of Def. 3.1 then implies the Novikov condition for expression (1) and at the same time that expression (2) belongs to $L^1(P)$. So we have that $Z_t \in L^2(P)$.

To guarantee that $R_t \in L^2(P)$, we need that

$$\mathbb{E} \left[\int_t^T e^{8(1-\gamma)\left[(2(1-\gamma)-\frac{1}{2})+rs+\int_0^s \pi_u^P \lambda^P \sigma^P du + \int_0^s \pi_u^S \lambda_u \sigma_u^* du\right]} ds \right] < \infty.$$

This condition is then implied by conditions (3.5) and (3.6) of Def. 3.1, where we note that $8(1-\gamma) < 0$ for $\gamma > 1$.

Now (3.30) is proved and since (3.29) holds, we have finally fulfilled condition (3.28).

Part 2: $\gamma = 1$.

Just using analogous arguments as in the power utility case we arrive at

$$\mathbb{E}^{t,v} [\log(V_T^{\pi^*})] - \mathbb{E}^{t,v} \left[\int_t^T \kappa \frac{(\lambda_u^*)^\alpha}{\alpha} du \right] = J'(t, v; \pi^*, \lambda^*) = \Phi(t, v).$$

The equation corresponding to (3.27) for the log utility case then reads:

$$\Phi(T, V_T^\pi) - \int_t^T \kappa \frac{\lambda_u^\alpha}{\alpha} du \leq \Phi(t, v) + \int_t^T \pi_u^P \sigma^P dW_u^P + \int_t^T \pi_u^S \sigma_u^* dW_u,$$

where we have used that $\Phi_v(t, v) = 1/v$. Taking the expectation on both both sides and keeping in mind that $\Phi(t, v) = \log(v)$ then yields

$$\begin{aligned} \mathbb{E}^{t,v} [\log(V_T^\pi)] - \mathbb{E}^{t,v} \left[\int_t^T \kappa \frac{(\lambda_u)^\alpha}{\alpha} du \right] &= J'(t, v; \pi, \lambda) \\ &\leq \Phi(t, v) + \underbrace{\mathbb{E}^{t,v} \left[\int_t^T \pi_u^P \sigma^P dW_u^P + \int_t^T \pi_u^S \sigma_u^* dW_u \right]}_{=0, \text{ by (3.4)}}. \end{aligned}$$

□

4 Discussion and Implications of Results

Our results in Theorem 3.1, Theorem 3.2, and Theorem 3.3 indicate the unconstrained executive's maximized utility and his behavior regarding personal portfolio selection and choice of work effort in the constant relative risk aversion setup. Subsequently we investigate the sensitivity of these optimal strategies when varying the characteristics of the executive: risk aversion, productivity, and disutility stress. Additionally, we derive the fair compensation for the work effort of the executive in an indifference utility framework, see, e.g., Lambert, Larcker and Verecchia (1991) for a related approach.

The executive is characterized by the relative risk aversion coefficient ($\gamma > 0$), and two work effectiveness parameters: productivity ($1/\kappa$, with $\kappa > 0$), and disutility stress (α , with $\alpha > 2$). To produce results that have relativity to a base-level of work effort (i.e. a base-level control strategy specified by the Sharpe ratio λ_0), the disutility c^* is reparameterized so that the set-up becomes

$$U(v) = \begin{cases} \frac{v^{1-\gamma}}{1-\gamma}, & \text{for } \gamma > 0 \text{ and } \gamma \neq 1 \\ \log(v), & \text{for } \gamma = 1 \end{cases}$$

and

$$c^*(t, v, \lambda) = \frac{\kappa}{\alpha} v^{1-\gamma} \left(\frac{\lambda}{\lambda_0} \right)^\alpha, \quad \text{for } \lambda \geq 0, \quad \gamma > 0.$$

For most parts of this section we focus on the optimal effort choice (λ^*), and pass over the optimal portfolio strategy (π^*). The optimal own-company stockholding (π^{S^*}) is a function of the optimal effort choice and of the optimized volatility (σ^* , derived in Lemma 2.1), that is not explicitly specified. The optimal holdings in the market portfolio (π^{P^*}) coincides with the result from classical utility maximization in the constant relative risk aversion setting, and is therefore of limited interest.

4.1 The Log-Utility Case

The optimal choice of effort in the new parameterizations is $\lambda^* = \lambda_0^{\frac{\alpha}{\alpha-2}} \kappa^{-\frac{1}{\alpha-2}}$ (see Theorem 3.2 for the optimal solution in the original parametrization). We require a minimal productive efficiency, that is $\kappa^{-1} > \lambda_0^{-2}$, to ensure that the optimal work effort is greater than the corresponding base level, i.e. $\lambda^* \geq \lambda_0$. Under this assumption we can formulate the optimal work effort as a function of productivity and disutility stress, $\lambda^* = \lambda^*(\kappa, \alpha)$, and calculate the sensitivities

$$\frac{\partial \lambda^*}{\partial(1/\kappa)} = \frac{1/\kappa}{\alpha-2} \lambda^* > 0, \quad \text{and} \quad \frac{\partial \lambda^*}{\partial \alpha} = \lambda^* \frac{\ln \kappa / \lambda_0^2}{(\alpha-2)^2} < 0, \quad \text{for } \alpha > 2, \text{ and } \kappa^{-1} > \lambda_0^{-2}.$$

The executive's optimal work effort choice is positively related to his work productivity parameter ($\frac{\partial \lambda^*}{\partial(1/\kappa)} > 0$), and negatively related to his disutility stress parameter ($\frac{\partial \lambda^*}{\partial \alpha} < 0$).

Figure 1 depicts the optimal effort choice (λ^*) as a function of work productivity ($1/\kappa$) and disutility stress (α), with $\lambda_0 = 0.10$. As predicted, the optimal effort choice, the idiosyncratic Sharpe ratio (λ^*), increases with work productivity ($1/\kappa$) and decreases with disutility stress (α). For moderate and large values of disutility stress, the optimal effort appears to be mainly driven by the executive's productivity where we observe a higher sensitivity for the productivity ($1/\kappa$) being close to the designated boundary value ($1/\lambda_0^2 = 100$).

Interesting limiting cases are:

$$\lim_{\kappa \nearrow \lambda_0^2} \lambda^*(\kappa, \alpha) = \lambda_0 \quad \text{and} \quad \lim_{\kappa \searrow 0} \lambda^*(\kappa, \alpha) = +\infty, \quad \text{for all } \alpha > 2,$$

indicating that the limit for deteriorating work productivity is base-level work effort (λ_0), and ever increasing work productivity yields ever increasing effort (to infinity). Taking the disutility stress parameter to its boundary cases gives

$$\lim_{\alpha \searrow 2} \lambda^*(\kappa, \alpha) = +\infty \quad \text{and} \quad \lim_{\alpha \nearrow \infty} \lambda^*(\kappa, \alpha) = \lambda_0, \quad \text{for all } \kappa^{-1} > \lambda_0^{-2},$$

indicating that the executive will deliver ever increasing work effort as disutility stress diminishes ($\alpha \searrow 2$), and the totally stressed executive ($\alpha \nearrow \infty$) will deliver base-level effort.

The executive's maximized utility from his optimal personal investment and work effort decision can be written as difference of the utility from investment and disutility from effort:

$$\Phi(v, 0) = \underbrace{\log v + \left[r + \frac{1}{2} (\lambda^P)^2 + \frac{1}{2} (\lambda^*)^2 \right] T}_{=\mathbb{E}^{0,v}[U(V_T^{\pi^*})]} - \underbrace{\frac{1}{\alpha} (\lambda^*)^2 T}_{\mathbb{E}^{0,v} \int_0^T c(\lambda^*(t, V_t^{\pi^*})) dt}.$$

Applying indifference utility arguments, the executive's fair compensation for the cost of effort can be paid as an upfront cash compensation (Δv). Then the fair compensation is a cash upfront payment (Δv) that is the solution of:

$$\Phi(v + \Delta v, 0) = \Phi(v, 0) + \mathbb{E}^{0,v} \left[\int_0^T c(\lambda^*(t, V_t^{\pi^*})) dt \right] \quad (4.1)$$

The solution is

$$\Delta v = v \left(e^{\frac{(\lambda^*)^2 T}{\alpha}} - 1 \right) = v \left(e^{\frac{\lambda_0^2 T}{\alpha} \left(\frac{\lambda_0^2}{\kappa} \right)^{\frac{2}{\alpha-2}}} - 1 \right).$$

The sensitivities of the compensation ($\Delta v(\kappa, \alpha)$) with respect to changes in the work productivity and disutility stress parameters are as expected:

$$\frac{\partial \Delta v}{\partial (1/\kappa)} = \frac{2}{\alpha - 2} \frac{1}{\kappa} \frac{(\lambda^*)^2}{\alpha} \Delta v > 0, \quad \text{for } \alpha > 2 \text{ and } \kappa^{-1} > \lambda_0^{-2}.$$

and

$$\frac{\partial \Delta v}{\partial \alpha} = - \left(\frac{1}{\alpha} + \frac{2 \ln \lambda_0^2 / \kappa}{(\alpha - 2)^2} \right) \frac{(\lambda^*)^2}{\alpha} \Delta v < 0, \quad \text{for } \alpha > 2 \text{ and } \kappa^{-1} > \lambda_0^{-2}.$$

The executive's indifference utility compensation therefore increases with his work productivity and decreases with his disutility stress.

Figure 2 displays a graph of the the fair cash up-front compensation based on the indifference utility rationale. The fair compensation (Δv) depends on the executive's initial wealth (v), the specific time horizon (T), and the base-level Sharpe ration (λ_0). For the present case we have chosen $v = \$5$ Mio, $T = 10$ years, and $\lambda_0 = 0.10$. Interesting limiting cases are:

$$\lim_{\kappa \nearrow \lambda_0^2} \Delta v(\kappa, \alpha) = v \left(e^{\frac{\lambda_0^2 T}{\alpha}} - 1 \right), \quad \text{and} \quad \lim_{\kappa \searrow 0} \Delta v(\kappa, \alpha) = +\infty, \quad \text{for all } \alpha > 2,$$

indicating that the limit for deteriorating work productivity to base-level work effort ($1/\lambda_0^2$) is the corresponding fair compensation, and ever increasing work productivity yields ever increasing fair compensation (to infinity). Taking the disutility stress parameter to its boundary cases gives

$$\lim_{\alpha \searrow 2} \Delta v(\kappa, \alpha) = +\infty, \quad \text{and} \quad \lim_{\alpha \nearrow \infty} \Delta v(\kappa, \alpha) = v \left(e^{\frac{\lambda_0^2 T}{\alpha}} - 1 \right), \quad \text{for all } \kappa < \lambda_0^2,$$

indicating that the executive will receive ever increasing fair compensation as disutility stress diminishes ($\alpha \searrow 2$), and the totally stressed executive ($\alpha \nearrow \infty$) will receive the base-level compensation (case: $\lambda^* = \lambda_0$).

4.2 The Power-Utility Case

The optimal effort in the new parametrization reads $\lambda^*(t) = \lambda_0^{\frac{\alpha}{\alpha-2}} (\kappa \gamma)^{-\frac{1}{\alpha-2}} f(t)^{\frac{1}{\alpha-2}}$, where we have dropped the dependence on the variable v . To ensure that the optimal effort is greater than the base effort we assume $r > -\frac{1}{2} \frac{\lambda_P^2}{\gamma}$ and

$$\kappa^{-1} \geq \begin{cases} \gamma \lambda_0^{-2}, & \text{for } 0 < \gamma < 1, \\ \gamma \lambda_0^{-2} f(0)^{-1}, & \text{for } \gamma > 1. \end{cases}$$

The conditions above follow from properties of the function f that is a solution of an ordinary differential equation of Bernoulli-type. Also note that in the new parametrization $f(0)$ reads

$$f(0) = e^{(1-\gamma) \left(r + \frac{1}{2} \frac{\lambda_P^2}{\gamma} \right) T} \left(1 - \frac{(\alpha - 2) \left(\frac{\lambda_0^\alpha}{\kappa \gamma} \right)^{\frac{2}{\alpha-2}}}{\alpha (2\gamma r + \lambda_P^2)} \left(e^{\frac{1-\gamma}{\alpha-2} \left(2r + \frac{\lambda_P^2}{\gamma} \right) T} - 1 \right) \right)^{-\frac{\alpha-2}{2}}.$$

The executive's fair compensation is derived from Equation (4.1). The cash upfront payment Δv is

$$\Delta v = v \left(e^{\frac{1}{2\gamma} \int_0^T \lambda^*(t)^2 dt} \left[1 - \frac{(\alpha - 2) \left(\frac{\lambda_0^\alpha}{\kappa \gamma} \right)^{\frac{2}{\alpha-2}}}{\alpha (2\gamma r + \lambda_P^2)} \left(e^{\frac{1-\gamma}{\alpha-2} \left(2r + \frac{\lambda_P^2}{\gamma} \right) T} - 1 \right) \right]^{\frac{(\alpha-2)}{2(1-\gamma)}} - 1 \right).$$

Remark 4.1. *The presented solution Δv is derived by using structural properties of the optimal portfolio strategies. The optimal portfolio strategy π^* is identical to that of an outsider investor $\hat{\pi}^*$ with knowledge of the effort exercised by the executive, that is, the outside investor knows λ^* . Denote $\hat{\Phi}(v, 0)$ the maximized utility of the outside investor, then it follows that $\hat{\Phi}(v, 0) = \Phi(v, 0) + \mathbb{E}[\int_0^T c^*(\lambda^*(t)) dt]$. Further, we can calculate*

$$\hat{\Phi}(0, v) = \frac{v^{1-\gamma}}{1-\gamma} e^{(1-\gamma) \left[\left(r + \frac{1}{2} \frac{\lambda_P^2}{\gamma} \right) T + \frac{1}{2\gamma} \int_0^T \lambda^*(t)^2 dt \right]},$$

and then solve $\hat{\Phi}(v, 0) = \Phi(v + \Delta, 0)$, the indifference utility principle as given in Equation (4.1), for Δv .

In contrast to the log-utility case, the sensitivities of the optimal effort λ^* and the fair compensation Δv with respect to variations in the parameters describing the executive cannot be given in a compact expression.

First the sensitivities of the optimal work effort (λ^*) is investigated. Figure 3 displays the optimal effort (λ^*) over time for varying risk aversion γ . It is notable that an executive with a rather low risk aversion ($0 < \gamma < 1$) starts at a high effort level and then decreases over time. For an executive with a rather high risk aversion ($\gamma > 1$) the effort starts at a lower level and increases over time. Observing the executive's effort over time therefore potentially reveals his risk aversion. Figure 4 is in line with the previous observation. The risk aversion is fixed at a rather low value ($\gamma = 0.5$), hence the executive's effort decreases over time. The executive's effort (λ^*) increases with his productivity (κ^{-1}) indicating that a more productive executive will work harder in order to benefit from the company's stock price growth through his investment decision of own-company stockholding. Figure 5 is similar to the previous setting, but now the disutility stress (α) is varied, with productivity being fixed ($\kappa^{-1} = 2000$). Increasing susceptibility to stress leads to decreasing effort.

The executive's fair compensation is now analyzed. The sensitivities of the upfront cash payment (Δv) is studied with respect to variations in the parameters describing the executive. Figure 6 shows the fair compensation graphed against productivity (κ^{-1}) and risk aversion (γ), disutility stress fixed ($\alpha = 5$). Decreasing risk aversion and increasing productivity leads to an increasing compensation. This effect becomes more notable for executives with a rather low risk aversion ($\gamma \approx 0.5$ and below). In Figure 7 the executives risk aversion is fixed ($\gamma = 0.5$) and the other parameters vary. Increasing productivity (κ^{-1}) and decreasing disutility stress (α) leads to an increasing fair compensation, where the relationship is more sensitive for small values of disutility stress (α). In Figure 8 the

executive's productivity is fixed ($\kappa^{-1} = 2000$), and risk aversion (γ) and disutility stress (α) are varied. The sensitivities are as observed before, and the effect becomes more pronounced for a rather low risk aversion ($\gamma \approx 0.5$ and below) and a rather low disutility stress ($\alpha \approx 4.5$). For even lower disutility stress ($\alpha \approx 4$ and below) the fair compensation (Δv) increases rapidly (what cannot be shown in the present figure).

In our framework the executive's effort choice (λ^*) and fair compensation (Δv) depend sensibly on the executive's characteristics, risk aversion (γ), work productivity (κ^{-1}), and disutility stress (α). Consequential observations are that the executive's risk aversion can be backed out from the exercised effort monitored over time, and that a better qualified executive (more productivity κ^{-1} and less disutility stress α) leads not only to a better performance but also to a higher indifference utility compensation (Δv). Thus the unconstrained executive is rewarded twice for talent. First he receives a higher compensation as a direct reward. Second, he benefits from investing in his own-company stock what can be termed an indirect reward.

5 Conclusion and Outlook

We establish a model framework that gives insight into an unconstrained executive's own-company stockholding and work effort preferences. The executive is characterized by risk aversion and work effectiveness parameters. We demonstrate that an executive with superior work effectiveness (i.e. higher quality) will undertake more work effort for his company. Furthermore, depending on any change in the company's non-systematic volatility associated with the executive's work effort (i.e. control strategy), due to risk aversion a higher quality executive will not necessarily undertake a higher own-company stockholding.

For application to empirical data, our framework allows an executive quality measure to be backed-out from the observed own-company stockholdings of unconstrained executives (assuming knowledge of non-systematic company volatility). Alternatively, with assumption of executive quality and risk aversion, our framework allows identification of the deviation in own-company stockholding that results from constraining an executive with performance contracting. A future extension for our framework is to specify a constrained executive subject to an imposed own-company stockholding representative of performance contracting, and to contrast his work effort strategy with that of our unconstrained executive.

References

- [1] Bettis, J., J. Bizjak and M. Lemmon (2001) Managerial ownership, incentive contracting, and the use of zero-cost collars and equity swaps by corporate insiders. In: *Journal of Financial and Quantitative Analysis* **36:3**, 345-370.

- [2] Cadenillas, A., J. Cvitanić and F. Zapatero (2004) Leverage decision and manager compensation with choice of effort and volatility. In: *Journal of Financial Economics* **73:1**, 71-92.
- [3] Carpenter, J. (2000) Does option compensation increase managerial risk appetite? In: *Journal of Finance* **55:5**, 2311-2331.
- [4] Core, J. and D. Larcker (2002) Performance consequences of mandatory increases in executive stock ownership. In: *Journal of Financial Economics* **64:3**, 317-340.
- [5] Core, J., W. Guay and D. Larcker (2003) Executive equity compensation and incentives: A survey. In: *Economic Policy Review* **9**, 27-50.
- [6] Cvitanić, J. (2008) On managerial risk-taking incentives when compensation may be hedged against. *Working Paper*, Caltech, Pasadena.
- [7] Dittmann, I. and E. Maug (2007) Lower salaries and no options? On the optimal structure of executive pay. In: *Journal of Finance* **62:1**, 303-343.
- [8] Garvey, G. and T. Milbourn (2003) Incentive compensation when executives can hedge the market: Evidence of relative performance evaluation in the cross section. In: *Journal of Finance* **58:4**, 1557-1581.
- [9] Hall, B. and J. Liebman (1998) Are CEOs really paid like bureaucrats? In: *Quarterly Journal of Economics* **113:3**, 653-691.
- [10] Holmstrom, B. (1979) Moral hazard and observability. In: *Bell Journal of Economics* **10**, 74-91.
- [11] Jensen, M. and W. Meckling (1976) Theory of the firm: Managerial behavior, agency costs and ownership structure. In: *Journal of Financial Economics* **3:4**, 305-360.
- [12] Jensen, M. and K. Murphy (1990) Performance pay and top-management incentives. In: *Journal of Political Economy* **98:2**, 225-264.
- [13] Jin, L. (2002) CEO compensation, diversification and incentives. In: *Journal of Financial Economics* **66:1**, 29-63.
- [14] Korn, R. and E. Korn (2001) *Option pricing and portfolio optimization*, Oxford Universtiy Press, Oxford.
- [15] Korn, R. and H. Kraft (2008) Continuous-Time Delegated Portfolio Management with Homogeneous Expectations: Can an Agency Conflict be Avoided? In: *Financial Markets and Portfolio Management* **22:1**, 67-90.
- [16] Lambert, R., D. Larcker and R. Verrecchia (1991) Portfolio considerations in valuing executive compensation. In: *Journal of Accounting Research* **29:1**, 129-148.

- [17] Murphy, K. (1999) Executive compensation. In: *O. Ashenfelter and D. Card, eds., Handbook of labor economics, Vol. 3*, Amsterdam, North-Holland.
- [18] Ofek, E. and D. Yermack (2000) Taking stock: Equity-based compensation and the evolution of managerial ownership. In: *Journal of Finance* **55:3**, 1367-1384.
- [19] Ou-Yang, H. (2003) Optimal contracts in a continuous-time delegated portfolio management problem. In: *Review of Financial Studies* **16:1**, 173-208.
- [20] Ross, S. (1973) The economic theory of agency: The principal's problem. In: *American Economic Review* **63:2**, 134-139.
- [21] Ross, S. (2004) Compensation, incentives, and the duality of risk aversion and riskiness. In: *Journal of Finance* **59:1**, 207-225.

Figures

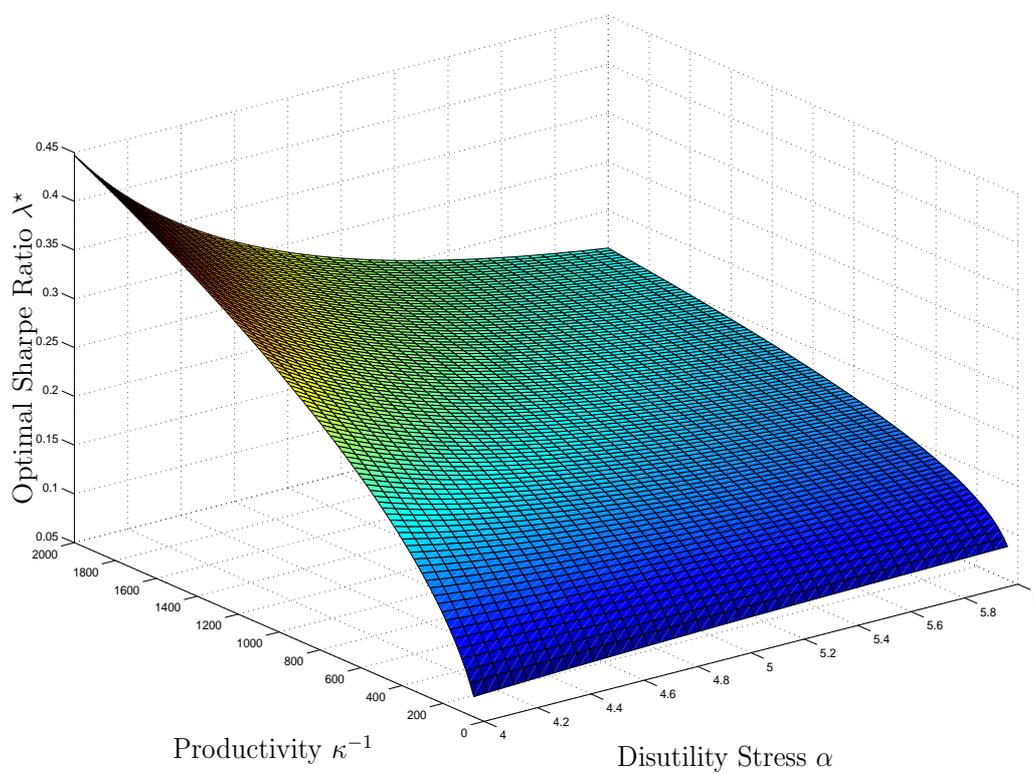


Figure 1: The optimal choice of the executive's effort λ^* is graphed against executive's characteristics work productivity κ^{-1} and disutility stress α ; with fixed base-level Sharpe ratio $\lambda_0 = 0.10$.

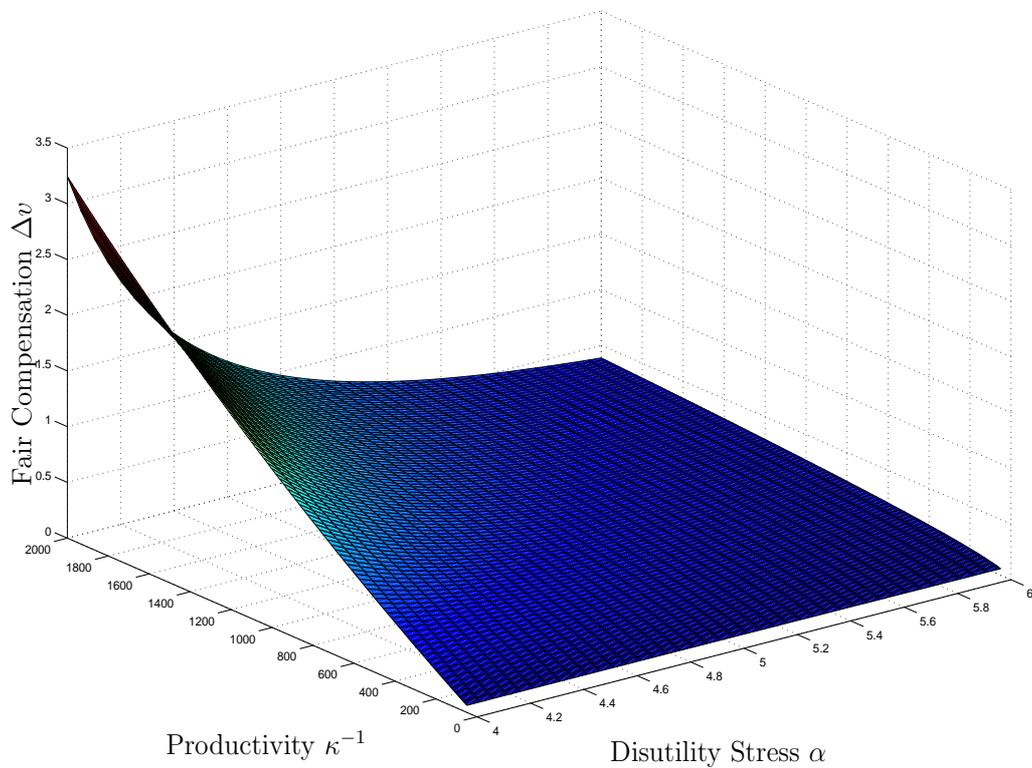


Figure 2: The executive's fair up-front cash compensation Δv (based on indifference utility) is graphed against executive's characteristics work productivity κ^{-1} and disutility stress α ; with fixed base-level Sharpe ratio $\lambda_0 = 0.10$, initial wealth $v = \$5$ Mio., and $T = 10$).

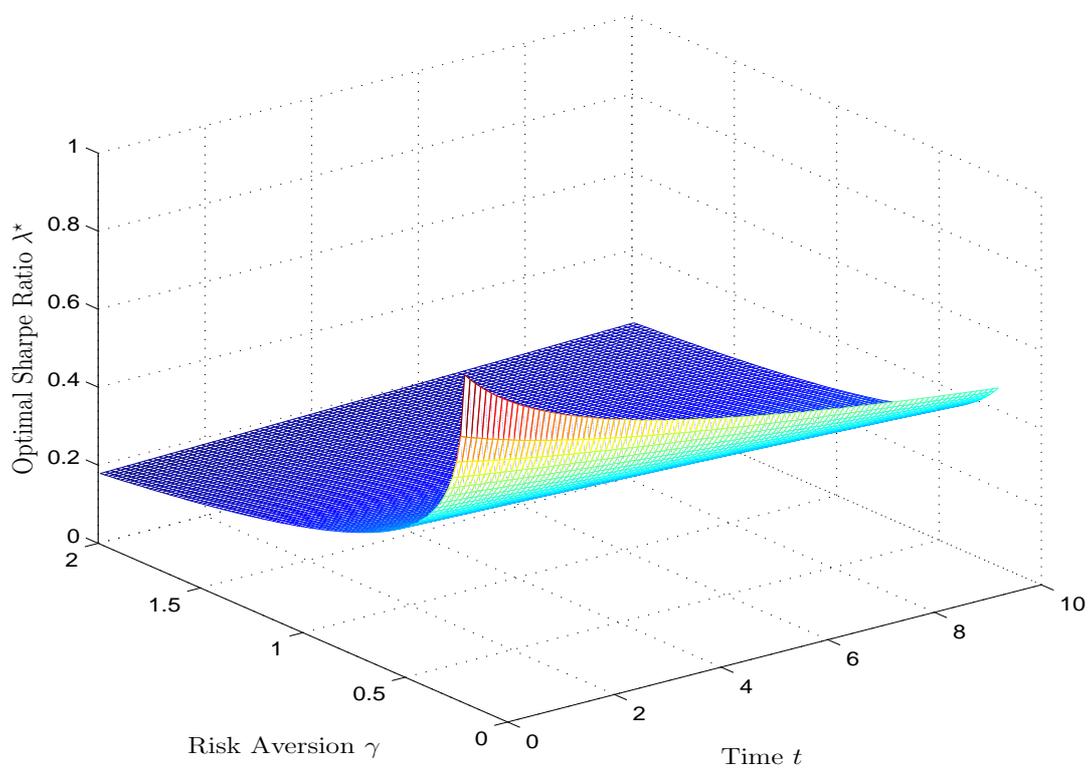


Figure 3: The optimal choice of the executive's effort λ^* is graphed against time t and risk-aversion γ ; with fixed base-level Sharpe ratio $\lambda_0 = 0.10$, disutility stress $\alpha = 5$, work productivity $\kappa^{-1} = 2000$.

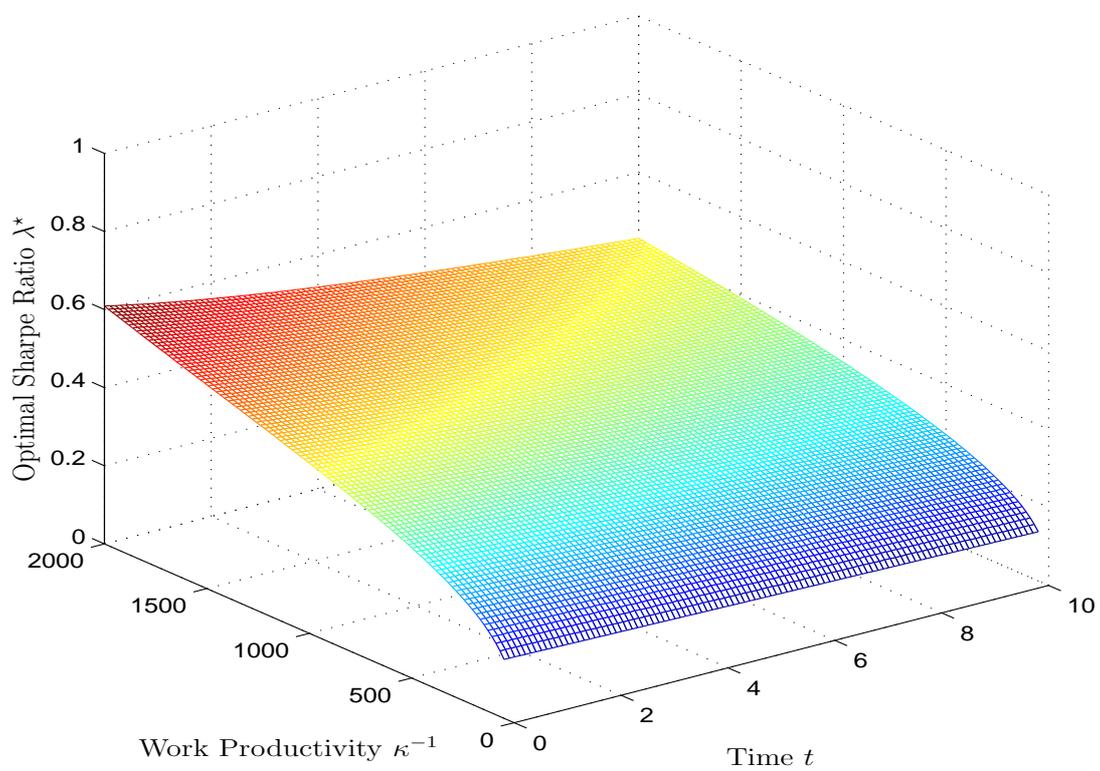


Figure 4: The optimal choice of the executive's effort λ^* is graphed against time t and work productivity κ^{-1} ; with fixed base-level Sharpe ratio $\lambda_0 = 0.10$, disutility stress $\alpha = 5$, risk aversion $\gamma = 0.25$.

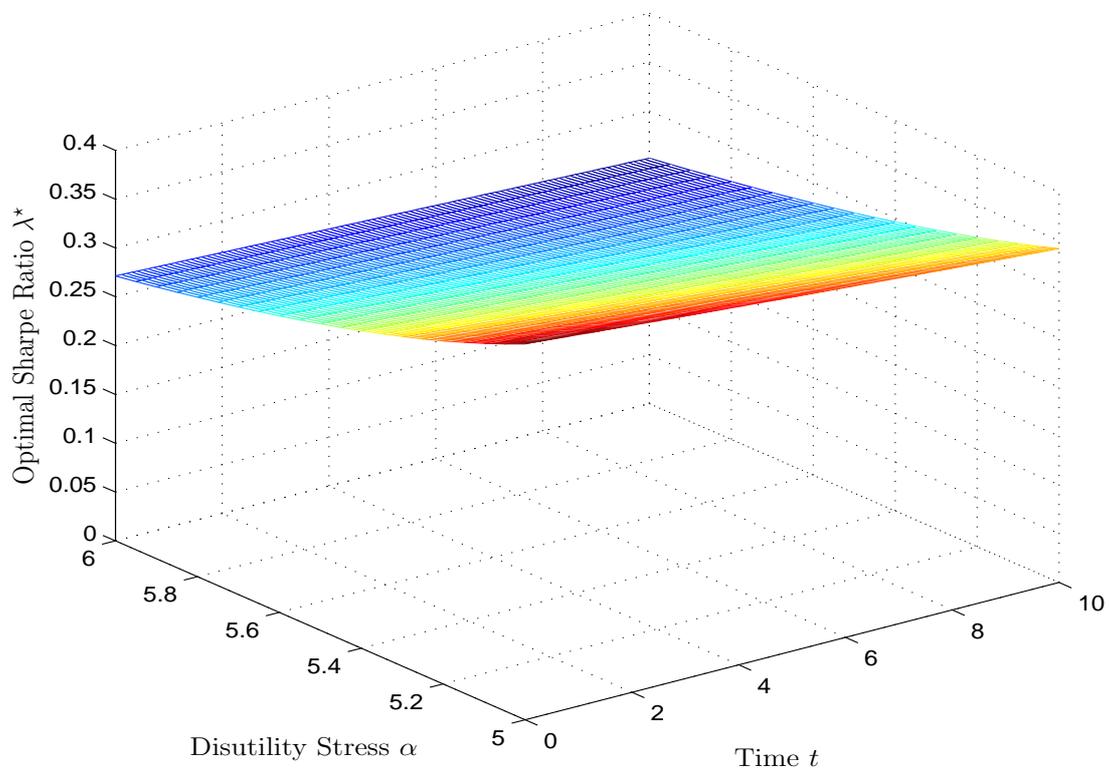


Figure 5: The optimal choice of the executive's effort λ^* is graphed against time t and disutility stress α ; with fixed base-level Sharpe ratio $\lambda_0 = 0.10$, work productivity $\kappa^{-1} = 2000$, risk aversion $\gamma = 0.5$.

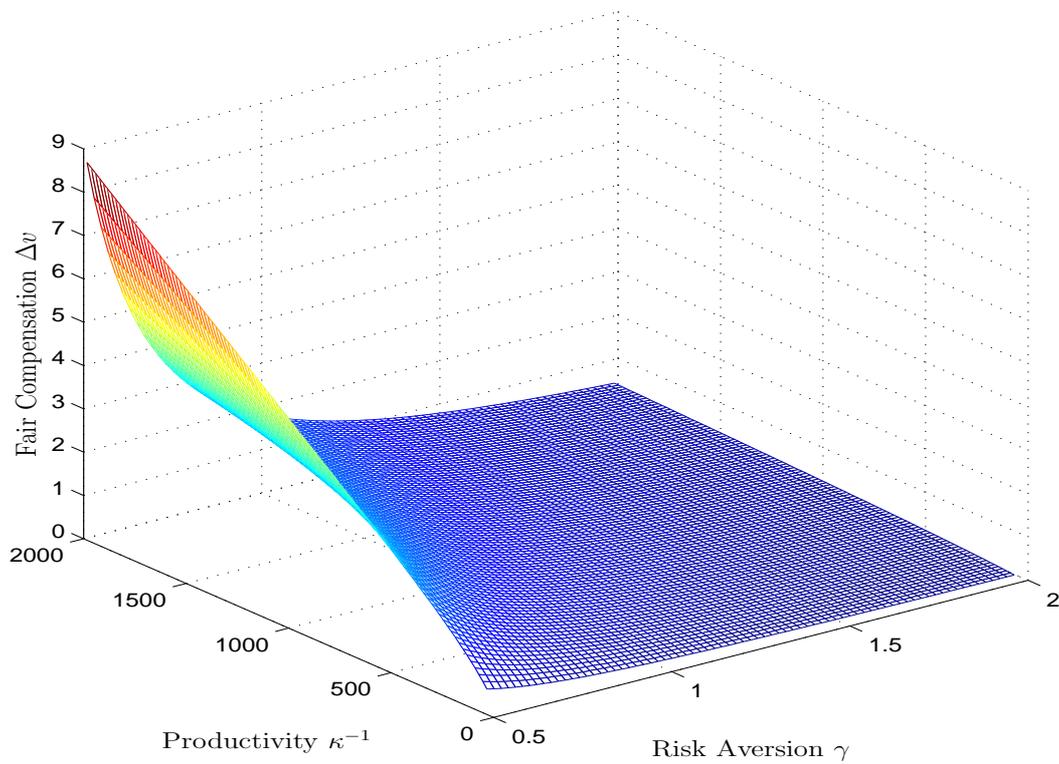


Figure 6: The executive's fair up-front cash compensation Δv (based on indifference utility) is graphed against the executive's characteristics work productivity κ^{-1} and risk aversion γ ; with fixed base-level Sharpe ratio $\lambda_0 = 0.10$, initial wealth $v = \$5$ Mio., $T = 10$, and disutility stress $\alpha = 5$.

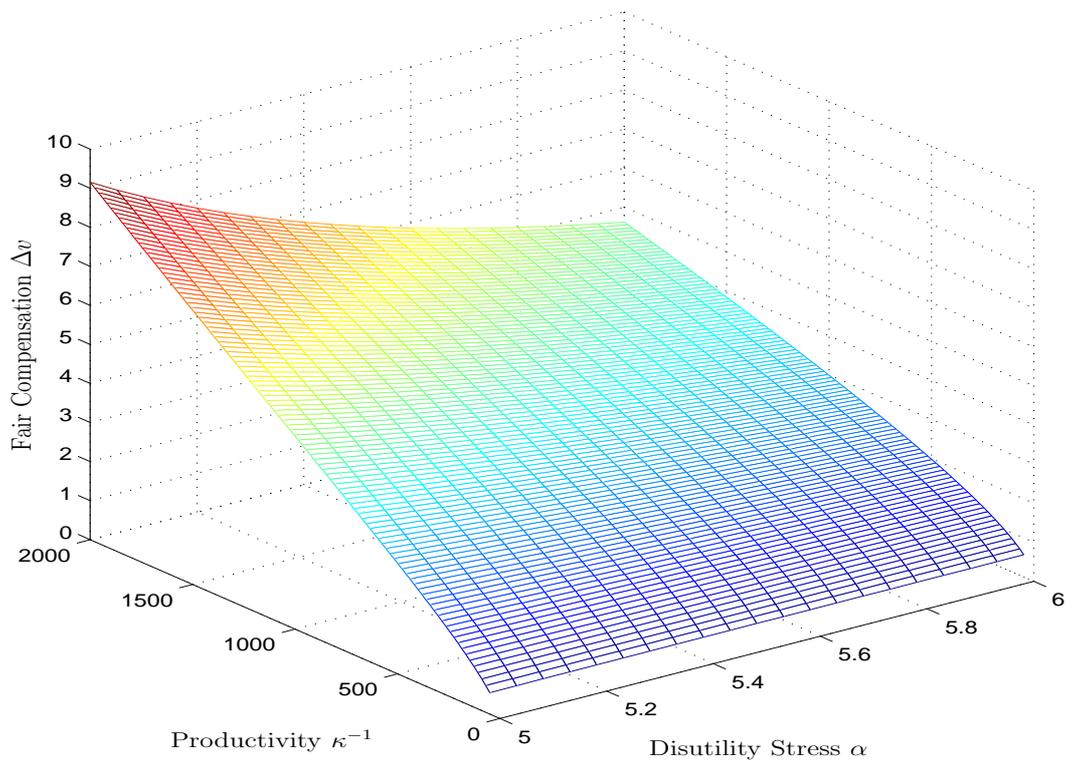


Figure 7: The executive's fair up-front cash compensation Δv (based on indifference utility) is graphed against the executive's characteristics work productivity κ^{-1} and disutility stress α ; with fixed base-level Sharpe ratio $\lambda_0 = 0.10$, initial wealth $v = \$5$ Mio., $T = 10$, and risk aversion $\gamma = 0.5$.

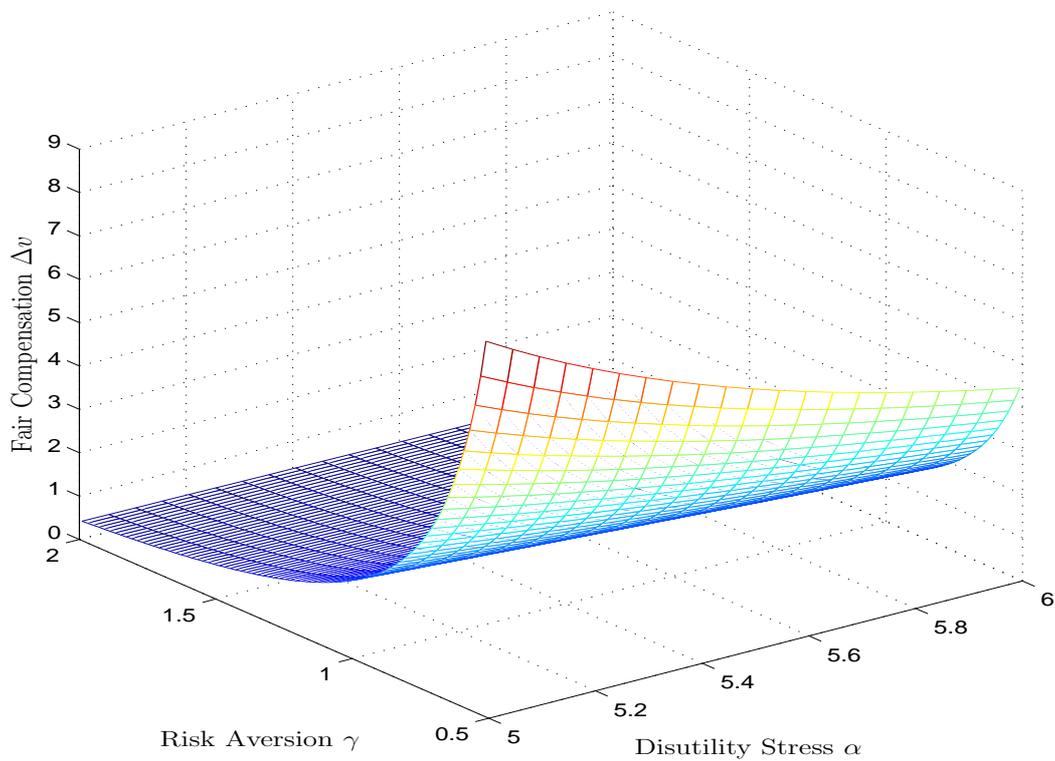


Figure 8: The executive's fair up-front cash compensation Δv (based on indifference utility) is graphed against the executive's characteristics risk aversion γ and disutility stress α ; with fixed base-level Sharpe ratio $\lambda_0 = 0.10$, initial wealth $v = \$5$ Mio., $T = 10$, and work productivity $\kappa^{-1} = 2000$.

Published reports of the Fraunhofer ITWM

The PDF-files of the following reports are available under:

www.itwm.fraunhofer.de/de/zentral__berichte/berichte

1. D. Hietel, K. Steiner, J. Struckmeier
A Finite - Volume Particle Method for Compressible Flows
(19 pages, 1998)
2. M. Feldmann, S. Seibold
Damage Diagnosis of Rotors: Application of Hilbert Transform and Multi-Hypothesis Testing
Keywords: Hilbert transform, damage diagnosis, Kalman filtering, non-linear dynamics
(23 pages, 1998)
3. Y. Ben-Haim, S. Seibold
Robust Reliability of Diagnostic Multi-Hypothesis Algorithms: Application to Rotating Machinery
Keywords: Robust reliability, convex models, Kalman filtering, multi-hypothesis diagnosis, rotating machinery, crack diagnosis
(24 pages, 1998)
4. F.-Th. Lentens, N. Siedow
Three-dimensional Radiative Heat Transfer in Glass Cooling Processes
(23 pages, 1998)
5. A. Klar, R. Wegener
A hierarchy of models for multilane vehicular traffic
Part I: Modeling
(23 pages, 1998)
Part II: Numerical and stochastic investigations
(17 pages, 1998)
6. A. Klar, N. Siedow
Boundary Layers and Domain Decomposition for Radiative Heat Transfer and Diffusion Equations: Applications to Glass Manufacturing Processes
(24 pages, 1998)
7. I. Choquet
Heterogeneous catalysis modelling and numerical simulation in rarified gas flows
Part I: Coverage locally at equilibrium
(24 pages, 1998)
8. J. Ohser, B. Steinbach, C. Lang
Efficient Texture Analysis of Binary Images
(17 pages, 1998)
9. J. Orlik
Homogenization for viscoelasticity of the integral type with aging and shrinkage
(20 pages, 1998)
10. J. Mohring
Helmholtz Resonators with Large Aperture
(21 pages, 1998)
11. H. W. Hamacher, A. Schöbel
On Center Cycles in Grid Graphs
(15 pages, 1998)
12. H. W. Hamacher, K.-H. Küfer
Inverse radiation therapy planning - a multiple objective optimisation approach
(14 pages, 1999)
13. C. Lang, J. Ohser, R. Hilfer
On the Analysis of Spatial Binary Images
(20 pages, 1999)
14. M. Junk
On the Construction of Discrete Equilibrium Distributions for Kinetic Schemes
(24 pages, 1999)
15. M. Junk, S. V. Raghurame Rao
A new discrete velocity method for Navier-Stokes equations
(20 pages, 1999)
16. H. Neunzert
Mathematics as a Key to Key Technologies
(39 pages (4 PDF-Files), 1999)
17. J. Ohser, K. Sandau
Considerations about the Estimation of the Size Distribution in Wicksell's Corpuscle Problem
(18 pages, 1999)
18. E. Carrizosa, H. W. Hamacher, R. Klein, S. Nickel
Solving nonconvex planar location problems by finite dominating sets
Keywords: Continuous Location, Polyhedral Gauges, Finite Dominating Sets, Approximation, Sandwich Algorithm, Greedy Algorithm
(19 pages, 2000)
19. A. Becker
A Review on Image Distortion Measures
Keywords: Distortion measure, human visual system
(26 pages, 2000)
20. H. W. Hamacher, M. Labbé, S. Nickel, T. Sonneborn
Polyhedral Properties of the Uncapacitated Multiple Allocation Hub Location Problem
Keywords: integer programming, hub location, facility location, valid inequalities, facets, branch and cut
(21 pages, 2000)
21. H. W. Hamacher, A. Schöbel
Design of Zone Tariff Systems in Public Transportation
(30 pages, 2001)
22. D. Hietel, M. Junk, R. Keck, D. Teleaga
The Finite-Volume-Particle Method for Conservation Laws
(16 pages, 2001)
23. T. Bender, H. Hennes, J. Kalcsics, M. T. Melo, S. Nickel
Location Software and Interface with GIS and Supply Chain Management
Keywords: facility location, software development, geographical information systems, supply chain management
(48 pages, 2001)
24. H. W. Hamacher, S. A. Tjandra
Mathematical Modelling of Evacuation Problems: A State of Art
(44 pages, 2001)
25. J. Kuhnert, S. Tiwari
Grid free method for solving the Poisson equation
Keywords: Poisson equation, Least squares method, Grid free method
(19 pages, 2001)
26. T. Götz, H. Rave, D. Reinel-Bitzer, K. Steiner, H. Tiemeier
Simulation of the fiber spinning process
Keywords: Melt spinning, fiber model, Lattice Boltzmann, CFD
(19 pages, 2001)
27. A. Zemitis
On interaction of a liquid film with an obstacle
Keywords: impinging jets, liquid film, models, numerical solution, shape
(22 pages, 2001)
28. I. Ginzburg, K. Steiner
Free surface lattice-Boltzmann method to model the filling of expanding cavities by Bingham Fluids
Keywords: Generalized LBE, free-surface phenomena, interface boundary conditions, filling processes, Bingham viscoplastic model, regularized models
(22 pages, 2001)
29. H. Neunzert
»Denn nichts ist für den Menschen als Menschen etwas wert, was er nicht mit Leidenschaft tun kann«
Vortrag anlässlich der Verleihung des Akademiepreises des Landes Rheinland-Pfalz am 21.11.2001
Keywords: Lehre, Forschung, angewandte Mathematik, Mehrskalalanalyse, Strömungsmechanik
(18 pages, 2001)
30. J. Kuhnert, S. Tiwari
Finite pointset method based on the projection method for simulations of the incompressible Navier-Stokes equations
Keywords: Incompressible Navier-Stokes equations, Meshfree method, Projection method, Particle scheme, Least squares approximation
AMS subject classification: 76D05, 76M28
(25 pages, 2001)
31. R. Korn, M. Krekel
Optimal Portfolios with Fixed Consumption or Income Streams
Keywords: Portfolio optimisation, stochastic control, HJB equation, discretisation of control problems
(23 pages, 2002)
32. M. Krekel
Optimal portfolios with a loan dependent credit spread
Keywords: Portfolio optimisation, stochastic control, HJB equation, credit spread, log utility, power utility, non-linear wealth dynamics
(25 pages, 2002)
33. J. Ohser, W. Nagel, K. Schladitz
The Euler number of discretized sets – on the choice of adjacency in homogeneous lattices
Keywords: image analysis, Euler number, neighborhood relationships, cuboidal lattice
(32 pages, 2002)

34. I. Ginzburg, K. Steiner
Lattice Boltzmann Model for Free-Surface flow and Its Application to Filling Process in Casting
Keywords: Lattice Boltzmann models; free-surface phenomena; interface boundary conditions; filling processes; injection molding; volume of fluid method; interface boundary conditions; advection-schemes; up-wind-schemes (54 pages, 2002)
35. M. Günther, A. Klar, T. Materne, R. Wegener
Multivalued fundamental diagrams and stop and go waves for continuum traffic equations
Keywords: traffic flow, macroscopic equations, kinetic derivation, multivalued fundamental diagram, stop and go waves, phase transitions (25 pages, 2002)
36. S. Feldmann, P. Lang, D. Prätzel-Wolters
Parameter influence on the zeros of network determinants
Keywords: Networks, Equicofactor matrix polynomials, Realization theory, Matrix perturbation theory (30 pages, 2002)
37. K. Koch, J. Ohser, K. Schladitz
Spectral theory for random closed sets and estimating the covariance via frequency space
Keywords: Random set, Bartlett spectrum, fast Fourier transform, power spectrum (28 pages, 2002)
38. D. d'Humières, I. Ginzburg
Multi-reflection boundary conditions for lattice Boltzmann models
Keywords: lattice Boltzmann equation, boundary conditions, bounce-back rule, Navier-Stokes equation (72 pages, 2002)
39. R. Korn
Elementare Finanzmathematik
Keywords: Finanzmathematik, Aktien, Optionen, Portfolio-Optimierung, Börse, Lehrerweiterbildung, Mathematikunterricht (98 pages, 2002)
40. J. Kallrath, M. C. Müller, S. Nickel
Batch Presorting Problems: Models and Complexity Results
Keywords: Complexity theory, Integer programming, Assignment, Logistics (19 pages, 2002)
41. J. Linn
On the frame-invariant description of the phase space of the Folgar-Tucker equation
Keywords: fiber orientation, Folgar-Tucker equation, injection molding (5 pages, 2003)
42. T. Hanne, S. Nickel
A Multi-Objective Evolutionary Algorithm for Scheduling and Inspection Planning in Software Development Projects
Keywords: multiple objective programming, project management and scheduling, software development, evolutionary algorithms, efficient set (29 pages, 2003)
43. T. Bortfeld, K.-H. Küfer, M. Monz, A. Scherrer, C. Thieke, H. Trinkaus
Intensity-Modulated Radiotherapy - A Large Scale Multi-Criteria Programming Problem
Keywords: multiple criteria optimization, representative systems of Pareto solutions, adaptive triangulation, clustering and disaggregation techniques, visualization of Pareto solutions, medical physics, external beam radiotherapy planning, intensity modulated radiotherapy (31 pages, 2003)
44. T. Halfmann, T. Wichmann
Overview of Symbolic Methods in Industrial Analog Circuit Design
Keywords: CAD, automated analog circuit design, symbolic analysis, computer algebra, behavioral modeling, system simulation, circuit sizing, macro modeling, differential-algebraic equations, index (17 pages, 2003)
45. S. E. Mikhailov, J. Orlik
Asymptotic Homogenisation in Strength and Fatigue Durability Analysis of Composites
Keywords: multiscale structures, asymptotic homogenization, strength, fatigue, singularity, non-local conditions (14 pages, 2003)
46. P. Domínguez-Marín, P. Hansen, N. Mladenović, S. Nickel
Heuristic Procedures for Solving the Discrete Ordered Median Problem
Keywords: genetic algorithms, variable neighborhood search, discrete facility location (31 pages, 2003)
47. N. Boland, P. Domínguez-Marín, S. Nickel, J. Puerto
Exact Procedures for Solving the Discrete Ordered Median Problem
Keywords: discrete location, Integer programming (41 pages, 2003)
48. S. Feldmann, P. Lang
Padé-like reduction of stable discrete linear systems preserving their stability
Keywords: Discrete linear systems, model reduction, stability, Hankel matrix, Stein equation (16 pages, 2003)
49. J. Kallrath, S. Nickel
A Polynomial Case of the Batch Presorting Problem
Keywords: batch presorting problem, online optimization, competitive analysis, polynomial algorithms, logistics (17 pages, 2003)
50. T. Hanne, H. L. Trinkaus
knowCube for MCDM – Visual and Interactive Support for Multicriteria Decision Making
Keywords: Multicriteria decision making, knowledge management, decision support systems, visual interfaces, interactive navigation, real-life applications. (26 pages, 2003)
51. O. Iliev, V. Laptev
On Numerical Simulation of Flow Through Oil Filters
Keywords: oil filters, coupled flow in plain and porous media, Navier-Stokes, Brinkman, numerical simulation (8 pages, 2003)
52. W. Dörfler, O. Iliev, D. Stoyanov, D. Vassileva
On a Multigrid Adaptive Refinement Solver for Saturated Non-Newtonian Flow in Porous Media
Keywords: Nonlinear multigrid, adaptive refinement, non-Newtonian flow in porous media (17 pages, 2003)
53. S. Kruse
On the Pricing of Forward Starting Options under Stochastic Volatility
Keywords: Option pricing, forward starting options, Heston model, stochastic volatility, cliquet options (11 pages, 2003)
54. O. Iliev, D. Stoyanov
Multigrid – adaptive local refinement solver for incompressible flows
Keywords: Navier-Stokes equations, incompressible flow, projection-type splitting, SIMPLE, multigrid methods, adaptive local refinement, lid-driven flow in a cavity (37 pages, 2003)
55. V. Starikovicus
The multiphase flow and heat transfer in porous media
Keywords: Two-phase flow in porous media, various formulations, global pressure, multiphase mixture model, numerical simulation (30 pages, 2003)
56. P. Lang, A. Sarishvili, A. Wirsen
Blocked neural networks for knowledge extraction in the software development process
Keywords: Blocked Neural Networks, Nonlinear Regression, Knowledge Extraction, Code Inspection (21 pages, 2003)
57. H. Knaf, P. Lang, S. Zeiser
Diagnosis aiding in Regulation Thermography using Fuzzy Logic
Keywords: fuzzy logic, knowledge representation, expert system (22 pages, 2003)
58. M. T. Melo, S. Nickel, F. Saldanha da Gama
Largescale models for dynamic multi-commodity capacitated facility location
Keywords: supply chain management, strategic planning, dynamic location, modeling (40 pages, 2003)
59. J. Orlik
Homogenization for contact problems with periodically rough surfaces
Keywords: asymptotic homogenization, contact problems (28 pages, 2004)
60. A. Scherrer, K.-H. Küfer, M. Monz, F. Alonso, T. Bortfeld
IMRT planning on adaptive volume structures – a significant advance of computational complexity
Keywords: Intensity-modulated radiation therapy (IMRT), inverse treatment planning, adaptive volume structures, hierarchical clustering, local refinement, adaptive clustering, convex programming, mesh generation, multi-grid methods (24 pages, 2004)
61. D. Kehrwald
Parallel lattice Boltzmann simulation of complex flows
Keywords: Lattice Boltzmann methods, parallel computing, microstructure simulation, virtual material design, pseudo-plastic fluids, liquid composite moulding (12 pages, 2004)
62. O. Iliev, J. Linn, M. Moog, D. Niedziela, V. Starikovicus
On the Performance of Certain Iterative Solvers for Coupled Systems Arising in Discretization of Non-Newtonian Flow Equations
Keywords: Performance of iterative solvers, Preconditioners, Non-Newtonian flow (17 pages, 2004)
63. R. Ciegis, O. Iliev, S. Rief, K. Steiner
On Modelling and Simulation of Different Regimes for Liquid Polymer Moulding
Keywords: Liquid Polymer Moulding, Modelling, Simulation, Infiltration, Front Propagation, non-Newtonian flow in porous media (43 pages, 2004)

64. T. Hanne, H. Neu
Simulating Human Resources in Software Development Processes
Keywords: Human resource modeling, software process, productivity, human factors, learning curve (14 pages, 2004)
65. O. Iliev, A. Mikelic, P. Popov
Fluid structure interaction problems in deformable porous media: Toward permeability of deformable porous media
Keywords: fluid-structure interaction, deformable porous media, upscaling, linear elasticity, stokes, finite elements (28 pages, 2004)
66. F. Gaspar, O. Iliev, F. Lisbona, A. Naumovich, P. Vabishchevich
On numerical solution of 1-D poroelasticity equations in a multilayered domain
Keywords: poroelasticity, multilayered material, finite volume discretization, MAC type grid (41 pages, 2004)
67. J. Ohser, K. Schladitz, K. Koch, M. Nöthe
Diffraction by image processing and its application in materials science
Keywords: porous microstructure, image analysis, random set, fast Fourier transform, power spectrum, Bartlett spectrum (13 pages, 2004)
68. H. Neunzert
Mathematics as a Technology: Challenges for the next 10 Years
Keywords: applied mathematics, technology, modelling, simulation, visualization, optimization, glass processing, spinning processes, fiber-fluid interaction, turbulence effects, topological optimization, multicriteria optimization, Uncertainty and Risk, financial mathematics, Malliavin calculus, Monte-Carlo methods, virtual material design, filtration, bio-informatics, system biology (29 pages, 2004)
69. R. Ewing, O. Iliev, R. Lazarov, A. Naumovich
On convergence of certain finite difference discretizations for 1D poroelasticity interface problems
Keywords: poroelasticity, multilayered material, finite volume discretizations, MAC type grid, error estimates (26 pages, 2004)
70. W. Dörfler, O. Iliev, D. Stoyanov, D. Vassileva
On Efficient Simulation of Non-Newtonian Flow in Saturated Porous Media with a Multigrid Adaptive Refinement Solver
Keywords: Nonlinear multigrid, adaptive refinement, non-Newtonian in porous media (25 pages, 2004)
71. J. Kalcsics, S. Nickel, M. Schröder
Towards a Unified Territory Design Approach – Applications, Algorithms and GIS Integration
Keywords: territory design, political districting, sales territory alignment, optimization algorithms, Geographical Information Systems (40 pages, 2005)
72. K. Schladitz, S. Peters, D. Reinel-Bitzer, A. Wiegmann, J. Ohser
Design of acoustic trim based on geometric modeling and flow simulation for non-woven
Keywords: random system of fibers, Poisson line process, flow resistivity, acoustic absorption, Lattice-Boltzmann method, non-woven (21 pages, 2005)
73. V. Rutka, A. Wiegmann
Explicit Jump Immersed Interface Method for virtual material design of the effective elastic moduli of composite materials
Keywords: virtual material design, explicit jump immersed interface method, effective elastic moduli, composite materials (22 pages, 2005)
74. T. Hanne
Eine Übersicht zum Scheduling von Baustellen
Keywords: Projektplanung, Scheduling, Bauplanung, Bauindustrie (32 pages, 2005)
75. J. Linn
The Folgar-Tucker Model as a Differential Algebraic System for Fiber Orientation Calculation
Keywords: fiber orientation, Folgar-Tucker model, invariants, algebraic constraints, phase space, trace stability (15 pages, 2005)
76. M. Speckert, K. Dreßler, H. Mauch, A. Lion, G. J. Wierda
Simulation eines neuartigen Prüfsystems für Achserprobungen durch MKS-Modellierung einschließlich Regelung
Keywords: virtual test rig, suspension testing, multibody simulation, modeling hexapod test rig, optimization of test rig configuration (20 pages, 2005)
77. K.-H. Küfer, M. Monz, A. Scherrer, P. Süß, F. Alonso, A. S. A. Sultan, Th. Bortfeld, D. Craft, Chr. Thieke
Multicriteria optimization in intensity modulated radiotherapy planning
Keywords: multicriteria optimization, extreme solutions, real-time decision making, adaptive approximation schemes, clustering methods, IMRT planning, reverse engineering (51 pages, 2005)
78. S. Amstutz, H. Andrä
A new algorithm for topology optimization using a level-set method
Keywords: shape optimization, topology optimization, topological sensitivity, level-set (22 pages, 2005)
79. N. Ettrich
Generation of surface elevation models for urban drainage simulation
Keywords: Flooding, simulation, urban elevation models, laser scanning (22 pages, 2005)
80. H. Andrä, J. Linn, I. Matei, I. Shklyar, K. Steiner, E. Teichmann
OPTCAST – Entwicklung adäquater Strukturoptimierungsverfahren für Gießereien Technischer Bericht (KURZFASSUNG)
Keywords: Topologieoptimierung, Level-Set-Methode, Gießprozesssimulation, Gießtechnische Restriktionen, CAE-Kette zur Strukturoptimierung (77 pages, 2005)
81. N. Marheineke, R. Wegener
Fiber Dynamics in Turbulent Flows Part I: General Modeling Framework
Keywords: fiber-fluid interaction; Cosserat rod; turbulence modeling; Kolmogorov's energy spectrum; double-velocity correlations; differentiable Gaussian fields (20 pages, 2005)
- Part II: Specific Taylor Drag**
Keywords: flexible fibers; $k-\epsilon$ turbulence model; fiber-turbulence interaction scales; air drag; random Gaussian aerodynamic force; white noise; stochastic differential equations; ARMA process (18 pages, 2005)
82. C. H. Lampert, O. Wirjadi
An Optimal Non-Orthogonal Separation of the Anisotropic Gaussian Convolution Filter
Keywords: Anisotropic Gaussian filter, linear filtering, orientation space, nD image processing, separable filters (25 pages, 2005)
83. H. Andrä, D. Stoyanov
Error indicators in the parallel finite element solver for linear elasticity DDFEM
Keywords: linear elasticity, finite element method, hierarchical shape functions, domain decomposition, parallel implementation, a posteriori error estimates (21 pages, 2006)
84. M. Schröder, I. Solchenbach
Optimization of Transfer Quality in Regional Public Transit
Keywords: public transit, transfer quality, quadratic assignment problem (16 pages, 2006)
85. A. Naumovich, F. J. Gaspar
On a multigrid solver for the three-dimensional Biot poroelasticity system in multilayered domains
Keywords: poroelasticity, interface problem, multigrid, operator-dependent prolongation (11 pages, 2006)
86. S. Panda, R. Wegener, N. Marheineke
Slender Body Theory for the Dynamics of Curved Viscous Fibers
Keywords: curved viscous fibers; fluid dynamics; Navier-Stokes equations; free boundary value problem; asymptotic expansions; slender body theory (14 pages, 2006)
87. E. Ivanov, H. Andrä, A. Kudryavtsev
Domain Decomposition Approach for Automatic Parallel Generation of Tetrahedral Grids
Key words: Grid Generation, Unstructured Grid, Delaunay Triangulation, Parallel Programming, Domain Decomposition, Load Balancing (18 pages, 2006)
88. S. Tiwari, S. Antonov, D. Hietel, J. Kuhnert, R. Wegener
A Meshfree Method for Simulations of Interactions between Fluids and Flexible Structures
Key words: Meshfree Method, FPM, Fluid Structure Interaction, Sheet of Paper, Dynamical Coupling (16 pages, 2006)
89. R. Ciegis, O. Iliev, V. Starikovicius, K. Steiner
Numerical Algorithms for Solving Problems of Multiphase Flows in Porous Media
Keywords: nonlinear algorithms, finite-volume method, software tools, porous media, flows (16 pages, 2006)
90. D. Niedziela, O. Iliev, A. Latz
On 3D Numerical Simulations of Viscoelastic Fluids
Keywords: non-Newtonian fluids, anisotropic viscosity, integral constitutive equation (18 pages, 2006)

91. A. Winterfeld
Application of general semi-infinite Programming to Lapidary Cutting Problems
Keywords: large scale optimization, nonlinear programming, general semi-infinite optimization, design centering, clustering
(26 pages, 2006)
92. J. Orlik, A. Ostrovska
Space-Time Finite Element Approximation and Numerical Solution of Hereditary Linear Viscoelasticity Problems
Keywords: hereditary viscoelasticity; kern approximation by interpolation; space-time finite element approximation, stability and a priori estimate
(24 pages, 2006)
93. V. Rutka, A. Wiegmann, H. Andrä
EJIM for Calculation of effective Elastic Moduli in 3D Linear Elasticity
Keywords: Elliptic PDE, linear elasticity, irregular domain, finite differences, fast solvers, effective elastic moduli
(24 pages, 2006)
94. A. Wiegmann, A. Zemitis
EJ-HEAT: A Fast Explicit Jump Harmonic Averaging Solver for the Effective Heat Conductivity of Composite Materials
Keywords: Stationary heat equation, effective thermal conductivity, explicit jump, discontinuous coefficients, virtual material design, microstructure simulation, EJ-HEAT
(21 pages, 2006)
95. A. Naumovich
On a finite volume discretization of the three-dimensional Biot poroelasticity system in multilayered domains
Keywords: Biot poroelasticity system, interface problems, finite volume discretization, finite difference method
(21 pages, 2006)
96. M. Krekel, J. Wenzel
A unified approach to Credit Default Swap-tion and Constant Maturity Credit Default Swap valuation
Keywords: LIBOR market model, credit risk, Credit Default Swaption, Constant Maturity Credit Default Swap-method
(43 pages, 2006)
97. A. Dreyer
Interval Methods for Analog Circuits
Keywords: interval arithmetic, analog circuits, tolerance analysis, parametric linear systems, frequency response, symbolic analysis, CAD, computer algebra
(36 pages, 2006)
98. N. Weigel, S. Weihe, G. Bitsch, K. Dreßler
Usage of Simulation for Design and Optimization of Testing
Keywords: Vehicle test rigs, MBS, control, hydraulics, testing philosophy
(14 pages, 2006)
99. H. Lang, G. Bitsch, K. Dreßler, M. Speckert
Comparison of the solutions of the elastic and elastoplastic boundary value problems
Keywords: Elastic BVP, elastoplastic BVP, variational inequalities, rate-independency, hysteresis, linear kinematic hardening, stop- and play-operator
(21 pages, 2006)
100. M. Speckert, K. Dreßler, H. Mauch
MBS Simulation of a hexapod based suspension test rig
Keywords: Test rig, MBS simulation, suspension, hydraulics, controlling, design optimization
(12 pages, 2006)
101. S. Azizi Sultan, K.-H. Küfer
A dynamic algorithm for beam orientations in multicriteria IMRT planning
Keywords: radiotherapy planning, beam orientation optimization, dynamic approach, evolutionary algorithm, global optimization
(14 pages, 2006)
102. T. Götz, A. Klar, N. Marheineke, R. Wegener
A Stochastic Model for the Fiber Lay-down Process in the Nonwoven Production
Keywords: fiber dynamics, stochastic Hamiltonian system, stochastic averaging
(17 pages, 2006)
103. Ph. Süß, K.-H. Küfer
Balancing control and simplicity: a variable aggregation method in intensity modulated radiation therapy planning
Keywords: IMRT planning, variable aggregation, clustering methods
(22 pages, 2006)
104. A. Beaudry, G. Laporte, T. Melo, S. Nickel
Dynamic transportation of patients in hospitals
Keywords: in-house hospital transportation, dial-a-ride, dynamic mode, tabu search
(37 pages, 2006)
105. Th. Hanne
Applying multiobjective evolutionary algorithms in industrial projects
Keywords: multiobjective evolutionary algorithms, discrete optimization, continuous optimization, electronic circuit design, semi-infinite programming, scheduling
(18 pages, 2006)
106. J. Franke, S. Halim
Wild bootstrap tests for comparing signals and images
Keywords: wild bootstrap test, texture classification, textile quality control, defect detection, kernel estimate, nonparametric regression
(13 pages, 2007)
107. Z. Drezner, S. Nickel
Solving the ordered one-median problem in the plane
Keywords: planar location, global optimization, ordered median, big triangle small triangle method, bounds, numerical experiments
(21 pages, 2007)
108. Th. Götz, A. Klar, A. Unterreiter, R. Wegener
Numerical evidence for the non-existing of solutions of the equations describing rotational fiber spinning
Keywords: rotational fiber spinning, viscous fibers, boundary value problem, existence of solutions
(11 pages, 2007)
109. Ph. Süß, K.-H. Küfer
Smooth intensity maps and the Bortfeld-Boyer sequencer
Keywords: probabilistic analysis, intensity modulated radiotherapy treatment (IMRT), IMRT plan application, step-and-shoot sequencing
(8 pages, 2007)
110. E. Ivanov, O. Gluchshenko, H. Andrä, A. Kudryavtsev
Parallel software tool for decomposing and meshing of 3d structures
Keywords: a-priori domain decomposition, unstructured grid, Delaunay mesh generation
(14 pages, 2007)
111. O. Iliev, R. Lazarov, J. Willems
Numerical study of two-grid preconditioners for 1d elliptic problems with highly oscillating discontinuous coefficients
Keywords: two-grid algorithm, oscillating coefficients, preconditioner
(20 pages, 2007)
112. L. Bonilla, T. Götz, A. Klar, N. Marheineke, R. Wegener
Hydrodynamic limit of the Fokker-Planck equation describing fiber lay-down processes
Keywords: stochastic differential equations, Fokker-Planck equation, asymptotic expansion, Ornstein-Uhlenbeck process
(17 pages, 2007)
113. S. Rief
Modeling and simulation of the pressing section of a paper machine
Keywords: paper machine, computational fluid dynamics, porous media
(41 pages, 2007)
114. R. Ciegis, O. Iliev, Z. Lakdawala
On parallel numerical algorithms for simulating industrial filtration problems
Keywords: Navier-Stokes-Brinkmann equations, finite volume discretization method, SIMPLE, parallel computing, data decomposition method
(24 pages, 2007)
115. N. Marheineke, R. Wegener
Dynamics of curved viscous fibers with surface tension
Keywords: Slender body theory, curved viscous bers with surface tension, free boundary value problem
(25 pages, 2007)
116. S. Feth, J. Franke, M. Speckert
Resampling-Methoden zur mse-Korrektur und Anwendungen in der Betriebsfestigkeit
Keywords: Weibull, Bootstrap, Maximum-Likelihood, Betriebsfestigkeit
(16 pages, 2007)
117. H. Knaf
Kernel Fisher discriminant functions – a concise and rigorous introduction
Keywords: wild bootstrap test, texture classification, textile quality control, defect detection, kernel estimate, nonparametric regression
(30 pages, 2007)
118. O. Iliev, I. Rybak
On numerical upscaling for flows in heterogeneous porous media
Keywords: numerical upscaling, heterogeneous porous media, single phase flow, Darcy's law, multiscale problem, effective permeability, multipoint flux approximation, anisotropy
(17 pages, 2007)
119. O. Iliev, I. Rybak
On approximation property of multipoint flux approximation method
Keywords: Multipoint flux approximation, finite volume method, elliptic equation, discontinuous tensor coefficients, anisotropy
(15 pages, 2007)
120. O. Iliev, I. Rybak, J. Willems
On upscaling heat conductivity for a class of industrial problems
Keywords: Multiscale problems, effective heat conductivity, numerical upscaling, domain decomposition
(21 pages, 2007)

121. R. Ewing, O. Iliev, R. Lazarov, I. Rybak
On two-level preconditioners for flow in porous media
Keywords: Multiscale problem, Darcy's law, single phase flow, anisotropic heterogeneous porous media, numerical upscaling, multigrid, domain decomposition, efficient preconditioner
(18 pages, 2007)
122. M. Brickenstein, A. Dreyer
POLYBORI: A Gröbner basis framework for Boolean polynomials
Keywords: Gröbner basis, formal verification, Boolean polynomials, algebraic cryptanalysis, satisfiability
(23 pages, 2007)
123. O. Wirjadi
Survey of 3d image segmentation methods
Keywords: image processing, 3d, image segmentation, binarization
(20 pages, 2007)
124. S. Zeytun, A. Gupta
A Comparative Study of the Vasicek and the CIR Model of the Short Rate
Keywords: interest rates, Vasicek model, CIR-model, calibration, parameter estimation
(17 pages, 2007)
125. G. Hanselmann, A. Sarishvili
Heterogeneous redundancy in software quality prediction using a hybrid Bayesian approach
Keywords: reliability prediction, fault prediction, non-homogeneous poisson process, Bayesian model averaging
(17 pages, 2007)
126. V. Maag, M. Berger, A. Winterfeld, K.-H. Küfer
A novel non-linear approach to minimal area rectangular packing
Keywords: rectangular packing, non-overlapping constraints, non-linear optimization, regularization, relaxation
(18 pages, 2007)
127. M. Monz, K.-H. Küfer, T. Bortfeld, C. Thieke
Pareto navigation – systematic multi-criteria-based IMRT treatment plan determination
Keywords: convex, interactive multi-objective optimization, intensity modulated radiotherapy planning
(15 pages, 2007)
128. M. Krause, A. Scherrer
On the role of modeling parameters in IMRT plan optimization
Keywords: intensity-modulated radiotherapy (IMRT), inverse IMRT planning, convex optimization, sensitivity analysis, elasticity, modeling parameters, equivalent uniform dose (EUD)
(18 pages, 2007)
129. A. Wiegmann
Computation of the permeability of porous materials from their microstructure by FFF-Stokes
Keywords: permeability, numerical homogenization, fast Stokes solver
(24 pages, 2007)
130. T. Melo, S. Nickel, F. Saldanha da Gama
Facility Location and Supply Chain Management – A comprehensive review
Keywords: facility location, supply chain management, network design
(54 pages, 2007)
131. T. Hanne, T. Melo, S. Nickel
Bringing robustness to patient flow management through optimized patient transports in hospitals
Keywords: Dial-a-Ride problem, online problem, case study, tabu search, hospital logistics
(23 pages, 2007)
132. R. Ewing, O. Iliev, R. Lazarov, I. Rybak, J. Willems
An efficient approach for upscaling properties of composite materials with high contrast of coefficients
Keywords: effective heat conductivity, permeability of fractured porous media, numerical upscaling, fibrous insulation materials, metal foams
(16 pages, 2008)
133. S. Gelareh, S. Nickel
New approaches to hub location problems in public transport planning
Keywords: integer programming, hub location, transportation, decomposition, heuristic
(25 pages, 2008)
134. G. Thömmes, J. Becker, M. Junk, A. K. Vainkuntam, D. Kehrwald, A. Klar, K. Steiner, A. Wiegmann
A Lattice Boltzmann Method for immiscible multiphase flow simulations using the Level Set Method
Keywords: Lattice Boltzmann method, Level Set method, free surface, multiphase flow
(28 pages, 2008)
135. J. Orlik
Homogenization in elasto-plasticity
Keywords: multiscale structures, asymptotic homogenization, nonlinear energy
(40 pages, 2008)
136. J. Almquist, H. Schmidt, P. Lang, J. Deitmer, M. Jirstrand, D. Prätzel-Wolters, H. Becker
Determination of interaction between MCT1 and CAII via a mathematical and physiological approach
Keywords: mathematical modeling; model reduction; electrophysiology; pH-sensitive microelectrodes; proton antenna
(20 pages, 2008)
137. E. Savenkov, H. Andrä, O. Iliev
An analysis of one regularization approach for solution of pure Neumann problem
Keywords: pure Neumann problem, elasticity, regularization, finite element method, condition number
(27 pages, 2008)
138. O. Berman, J. Kalcsics, D. Krass, S. Nickel
The ordered gradual covering location problem on a network
Keywords: gradual covering, ordered median function, network location
(32 pages, 2008)
139. S. Gelareh, S. Nickel
Multi-period public transport design: A novel model and solution approaches
Keywords: Integer programming, hub location, public transport, multi-period planning, heuristics
(31 pages, 2008)
140. T. Melo, S. Nickel, F. Saldanha-da-Gama
Network design decisions in supply chain planning
Keywords: supply chain design, integer programming models, location models, heuristics
(20 pages, 2008)
141. C. Lautensack, A. Särkkä, J. Freitag, K. Schladitz
Anisotropy analysis of pressed point processes
Keywords: estimation of compression, isotropy test, nearest neighbour distance, orientation analysis, polar ice, Ripley's K function
(35 pages, 2008)
142. O. Iliev, R. Lazarov, J. Willems
A Graph-Laplacian approach for calculating the effective thermal conductivity of complicated fiber geometries
Keywords: graph laplacian, effective heat conductivity, numerical upscaling, fibrous materials
(14 pages, 2008)
143. J. Linn, T. Stephan, J. Carlsson, R. Bohlin
Fast simulation of quasistatic rod deformations for VR applications
Keywords: quasistatic deformations, geometrically exact rod models, variational formulation, energy minimization, finite differences, nonlinear conjugate gradients
(7 pages, 2008)
144. J. Linn, T. Stephan
Simulation of quasistatic deformations using discrete rod models
Keywords: quasistatic deformations, geometrically exact rod models, variational formulation, energy minimization, finite differences, nonlinear conjugate gradients
(9 pages, 2008)
145. J. Marburger, N. Marheineke, R. Pinnau
Adjoint based optimal control using mesh-less discretizations
Keywords: Mesh-less methods, particle methods, Eulerian-Lagrangian formulation, optimization strategies, adjoint method, hyperbolic equations
(14 pages, 2008)
146. S. Desmettre, J. Gould, A. Szimayer
Own-company stockholding and work effort preferences of an unconstrained executive
Keywords: optimal portfolio choice, executive compensation
(33 pages, 2008)

Status quo: October 2008